CS/COE 1501

www.cs.pitt.edu/~lipschultz/cs1501/

String Pattern Matching

General idea

- Have a pattern string p of length m
- Have a text string a of length n
- Can we find an index i of string a such that each of the m characters in the substring of a starting at i matches each character in p
    - Example: can we find the pattern “fox” in the text “the quick brown fox jumps over the lazy dog”?
        - Yes! At index 16 of the text string!

Simple approach

- BRUTE FORCE
  - start at the beginning of both pattern and text
  - compare characters left to right
  - mismatch?
  - start again at the 2nd character of the text and the beginning of the pattern...

Brute force code

```
public static int bf_search(String pat, String txt) {
    int m = pat.length();
    int n = txt.length();
    for (int i = 0; i <= n - m; i++) {
        int j;
        for (j = 0; j < m; j++) {
            if (txt.charAt(i + j) != pat.charAt(j))
                break;
        }
        if (j == m)
            return i; // found at offset i
    }
    return n; // not found
}
```

Brute force analysis

- Runtime?
  - What does the worst case look like?
    - a = XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXY
    - p = XXXXY
  - m (n - m + 1)
    - Θ(nm) if n >> m
  - Is the average case runtime any better?
    - Assume we mostly miss on the first pattern character
    - Θ(n + m)
      - Θ(n) if n >> m

Where do we improve?

- Improve worst case
  - Theoretically very interesting
  - Practically doesn't come up that often for human language
- Improve average case
  - Much more practically helpful
    - Especially if we anticipate searching through large files

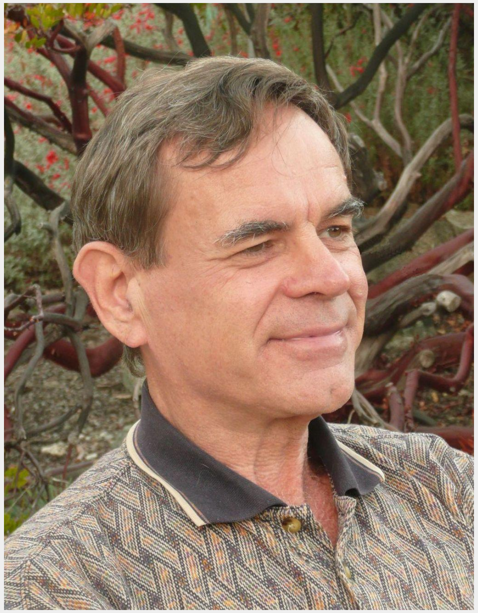
First: improving the worst case

Discovered the same algorithm independently

Knuth

Morris

Pratt



Worked together

Jointly published in 1976

First: improving the worst case

- Knuth Morris Pratt algorithm (KMP)
- Goal: avoid backing up in the text string on a mismatch
- Main idea: In checking the pattern, we learned something about the characters in the text, take advantage of this knowledge to avoid backing up

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text → A B A A A A B A A A A A A A A A

pattern → B A A A A A A A A A

What brute-force does when a mismatch is found.

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A B A A A A B A A A A A A A A A

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A B A A A A B A A A A A A A A A

B A A A A A A A A A

B A A A A A A A A A

Brute force
backs up to i+1

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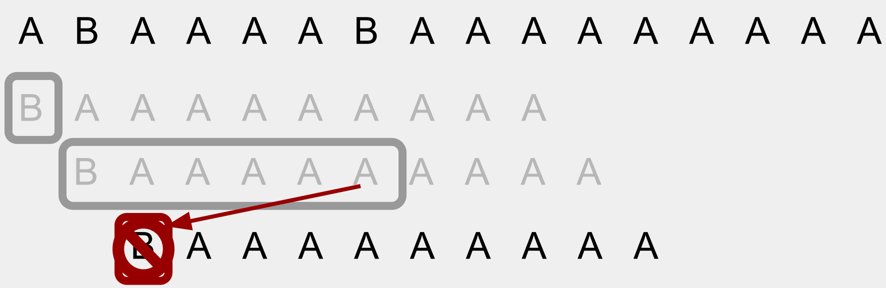


Brute force
backs up to i+1

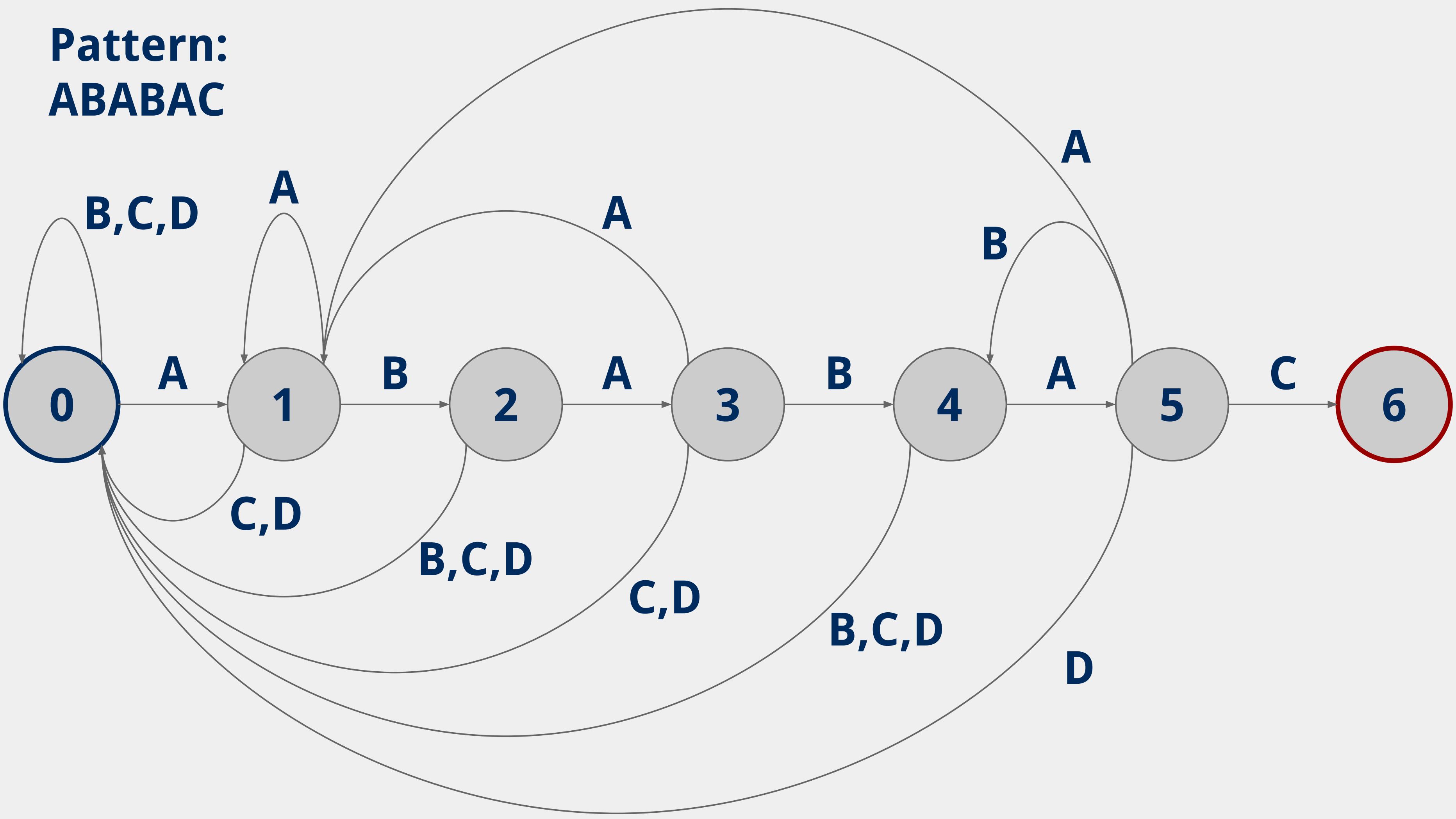
What brute-force does when a mismatch is found.

How do we keep track of text processed?

- Actually, build a deterministic finite-state automata (DFA) storing information about the *pattern*
  - From a given state in searching through the pattern, if you encounter a mismatch, how many characters currently match from the beginning of the pattern

DFA example

Pattern:
ABABAC



Representing the DFA in code

- DFA can be represented as a 2D array:
  - dfa[cur_text_char][pattern_counter] = new_pattern_counter
    - Storage needed?
      - mR

	0	1	2	3	4	5
A						
B						
C						
D						

KMP code

```
public int kmp_search(String pat, String txt) {
    int M = pat.length();
    int N = txt.length();
    int i, j;
    for (i = 0, j = 0; i < N && j < M; i++)
        j = dfa[txt.charAt(i)][j];
    if (j == M) return i - M; // found
    return N; // not found
}
```

- Runtime?

Another approach: Boyer Moore

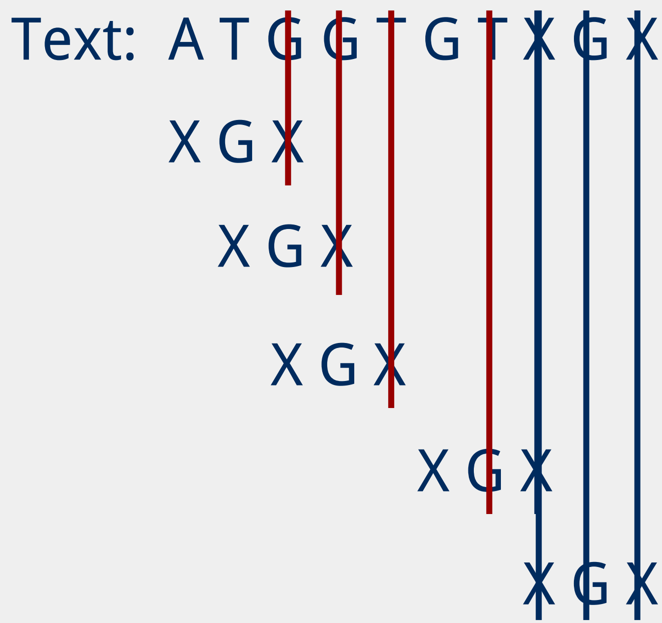
- What if we compare starting at the end of the pattern?
  - a = ABCDVABCDWABCDXABCDYABCDZ
  - p = ABCDE
  - V does not match E
    - Further V is nowhere in the pattern…
    - So skip ahead m positions with 1 comparison!
      - Runtime?
        - In the best case, n/m
- When searching through text with a large alphabet, will often come across characters not in the pattern.
  - One of Boyer Moore's heuristics takes advantage of this fact
    - Mismatched character heuristic

Missed character heuristic

- How well it works depends on the pattern and text at hand
  - What do we do in the general case after a mismatch?
    - Consider:
      - a = ATGGTGTXGX
      - p = XGX
    - If mismatched character *does* appear in p, need to "slide" to the right to the next occurrence of that character in p
      - Requires us to pre-process the pattern
        - Create a right array

```
for all i right[i] = -1;
for (int j = 0; j < m; j++)
    right[p.charAt(j)] = j;
```

Missed character heuristic example



Pattern: X G X

right = [-1, -1, …, 1, …, 2, ...]

G X

Runtime for missed character

- What does the worst case look like?
  - Runtime:
    - $\Theta(nm)$
      - Same as brute force!
- This is why missed character is only one of Boyer Moore’s heuristics
  - The works similarly to KMP
- See BoyerMoore.java

Another approach

- Hashing was cool, let's try using that

```
public static int hash_search(String pat, String txt) {
    int m = pat.length();
    int n = txt.length();
    int pat_hash = h(pat);
    for (int i = 0; i <= n - m; i++) {
        if (h(txt.substring(i, i + m)) == pat_hash)
            return i; // found!
    }
    return n; // not found
}
```

Well that was simple

- Is it efficient?
  - Nope! Practically worse than brute force
    - Instead of nm character comparisons, we perform n hashes of m character strings
- Can we make an efficient pattern matching algorithm based on hashing?

Horner's method

- Brought up during the hashing lecture

```
public long horners_hash(String key, int m) {
    long h = 0;
    for (int j = 0; j < m; j++)
        h = (R * h + key.charAt(j)) % Q;
    return h;
}
```

- horners_hash("abcd", 4) =
    - "a" * R^3 + "b" * R^2 + "c" * R + "d" mod 4
- What about horners_hash("bcde", 4)?

Rabin Karp

- Let a_i be a.charAt(i)
- Let x_i be $a_iR^{m-1} + a_{i+1}R^{m-2} + \ldots + a_{i+m-1}R^0$
- x_i mod Q == horners_hash(a.substring(i, i+m), m)
- x_{i+1} will then be: $(x_i - a_iR^{m-1})R + a_{i+m}$
- x_{i+1} mod Q == horners_hash(a.substring(i+1, i+m+1), m)
- Hence, we can avoid redoing a lot of hash recomputation

What about collisions?

- Note that we're not storing any values in a hash table…
  - So increasing Q doesn't affect memory utilization!
    - Make Q really big and the chance of a collision becomes really small!
      - But not 0…
- OK, so do a character by character comparison on a collision just to be sure
  - Worst case runtime?
    - Back to brute force esque runtime...

Assorted casinos

- Two options:
    - Do a character by character comparison after collision
        - Guaranteed correct
        - Probably fast

      Las Vegas

    - Assume a hash match means a substring match
        - Guaranteed fast
        - Probably correct

      Monte Carlo