

First and Follow Sets

Grammar

$E \rightarrow TX$
 $X \rightarrow + E$
 $X \rightarrow \epsilon$
 $T \rightarrow \text{int } Y$
 $T \rightarrow (E)$
 $Y \rightarrow * T$
 $Y \rightarrow \epsilon$

First Set

First(a) = set of terminals that start string of terminals derived from a.

Apply following rules until no terminal or ϵ can be added

1. If $t \in T$, then **First**(t) = { t }.
2. If $X \in N$ and $X \rightarrow \epsilon$ exists (nullable), then add ϵ to **First**(X).
3. If $X \in N$ and $X \rightarrow Y_1 Y_2 Y_3 \dots Y_m$, where $Y_1, Y_2, Y_3, \dots Y_m$ are non-terminals, then:

for each i from 1 to m

if $Y_1 \dots Y_{i-1}$ are all nullable (or if $i = 1$)

$$\mathbf{First}(X) = \mathbf{First}(X) \cup \mathbf{First}(Y_i)$$

By rule 1:

The **First** of a terminal is that terminal.

Symbol	First
((
))
+	+
*	*
int	int
Y	
X	
T	
E	

By rule 2:

If $X \in N$ and $X \rightarrow \epsilon$ exists (nullable), then add ϵ to **First**(X).

$X \rightarrow \epsilon$

$Y \rightarrow \epsilon$

Symbol	First
((
))
+	+
*	*
int	int
Y	ϵ
X	ϵ
T	
E	

By rule 3:

If $X \in N$ and $X \rightarrow Y_1 Y_2 Y_3 \dots Y_m$, where $Y_1, Y_2, Y_3, \dots, Y_m$ are non-terminals, then:

for each i from 1 to m

if $Y_1 \dots Y_{i-1}$ are all nullable (or if $i = 1$)

$$\mathbf{First}(X) = \mathbf{First}(X) \cup \mathbf{First}(Y_i)$$

$E \rightarrow T X$

$$\mathbf{First}(E) = \mathbf{First}(E) \cup \mathbf{First}(T)$$

We need First(T)... first.

$$\mathbf{First}(T) = \{ \text{int}, (\}$$

$T \rightarrow \text{int } Y$ because $\mathbf{First}(\text{int}) = \text{int}$

$T \rightarrow (E)$ because $\mathbf{First}(() = ($

Now we can go back and do **First**(E)

$$\mathbf{First}(E) = \{ \text{int}, (\}$$

We don't consider the first of X because T is not nullable.

$$\mathbf{First}(X) = \{ \epsilon, + \}$$

$X \rightarrow + E$ $\mathbf{First}(+) = +$

$X \rightarrow \epsilon$ handled at step 2

$\text{First}(Y) = \{ \epsilon, * \}$

$Y \rightarrow * T$ $\text{First}(*) = *$
 $Y \rightarrow \epsilon$ handled at step 2

Put it all together:

Symbol	First
((
))
+	+
*	*
int	int
Y	$\epsilon, *$
X	$\epsilon, +$
T	int, (
E	int, (

Done.

Follow Set

Apply following rules until no terminal or e can be added

1. $\$ \in \text{Follow}(S)$, where S is the start symbol.
2. Look at the occurrence of a non-terminal on the right hand side of a production which is followed by something
If $A \rightarrow a B b$, then $\text{First}(b) - \{\epsilon\} \subseteq \text{Follow}(B)$
3. Look at N on the RHS that is not followed by anything,
if $(A \rightarrow a B)$ or $(A \rightarrow a B b \text{ and } \epsilon \in \text{First}(b))$,
then $\text{Follow}(A) \subseteq \text{Follow}(B)$

By step 1:

$\$ \in \text{Follow}(S)$, where S is the start symbol.

a.) $\text{Follow}(E) = \{ \$ \}$ E is our start symbol

The table so far:

Symbol	First	Follow
((N/A
))	
+	+	
*	*	
int	int	
Y	$\epsilon, *$	
X	$\epsilon, +$	
T	int, (
E	int, (\$

By step 2:

Look at the occurrence of a non-terminal on the right hand side of a production which is followed by something:

If $A \rightarrow a B b$, then $\text{First}(b) - \{\epsilon\} \subseteq \text{Follow}(B)$

This subset notation can be confusing. It's saying here that the $\text{Follow}(B)$ contains at least the $\text{First}(b)$ (with ϵ excluded) as a subset. There might be more things in $\text{Follow}(B)$ because other steps added them in or other grammar rules were analyzed.

$E \rightarrow T X$

a.) $\text{Follow}(T)$ contains (at least) the $\text{First}(X) = \{\epsilon, +\} - \{\epsilon\} = \{+\}$

$T \rightarrow (E)$

b.) $\text{Follow}(E)$ contains (at least) the $\text{First}() = \{ \}$

So now $\text{Follow}(E)$ is $\{ \}, \$ \}$ (From step 1a)

The table so far:

Symbol	First	Follow
((N/A
))	
+	+	
*	*	
int	int	
Y	$\epsilon, *$	
X	$\epsilon, +$	
T	int, (+
E	int, (), \$

By step 3:

Look at N on the RHS that is not followed by anything,

if $(A \rightarrow a B)$ or $(A \rightarrow a B b \text{ and } \epsilon \in \text{First}(b))$,
then $\text{Follow}(A) \subseteq \text{Follow}(B)$

Again, be careful with the notation. It's saying here that the $\text{Follow}(B)$ contains at least the $\text{Follow}(A)$ as a subset. There might be more things in $\text{Follow}(B)$ because other steps added them in or other grammar rules were analyzed.

$E \rightarrow T X$

a.) $\text{Follow}(X)$ contains (at least) $\text{Follow}(E) = \{ \text{), } \$ \}$ (from step 2b)

$\epsilon \in \text{First}(X)$ so:

b.) $\text{Follow}(T)$ contains (at least) $\text{Follow}(E) = \{ \text{), } \$, + \}$ (from step 2a and 2b)

$X \rightarrow + E$

c.) $\text{Follow}(E)$ contains (at least) $\text{Follow}(X) = \{ \text{), } \$ \}$ (from step 3a)

$T \rightarrow \text{int } Y$

d.) $\text{Follow}(Y)$ contains (at least) $\text{Follow}(T) = \{ \text{), } \$, + \}$ (from step 3b)

$Y \rightarrow * T$

e.) $\text{Follow}(T)$ contains (at least) $\text{Follow}(Y) = \{ \text{), } \$ \}$ (from step 3d)

We do this whole process again until no more additions happen:

$E \rightarrow T X$

f.) $\text{Follow}(X)$ contains (at least) $\text{Follow}(E) = \{ \text{), } \$ \}$ (no change)

$\epsilon \in \text{First}(X)$ so:

g.) $\text{Follow}(T)$ contains (at least) $\text{Follow}(E) = \{ \text{), } \$, + \}$ (no change)

$X \rightarrow + E$

h.) $\text{Follow}(E)$ contains (at least) $\text{Follow}(X) = \{ \text{), } \$ \}$ (no change)

$T \rightarrow \text{int } Y$

i.) $\text{Follow}(Y)$ contains (at least) $\text{Follow}(T) = \{ \text{), } \$, + \}$ (no change)

$Y \rightarrow * T$

j.) $\text{Follow}(T)$ contains (at least) $\text{Follow}(Y) = \{ \text{), } \$ \}$ (no change)

Done.

Results

Symbol	First	Follow
((N/A
))	
+	+	
*	*	
int	int	
Y	$\epsilon, *$), \$, +
X	$\epsilon, +$), \$
T	int, (), \$, +
E	int, (), \$