Intutionistic Fuzzy-Based Method for Assessing the Learner's Knowledge Level and Personalization of Learning Path

Roya Hosseini[1], Ahmad Kardan[1]

(1) Advanced E-learning Technologies Laboratory (AELT)
Dept. of Computer Engineering and IT, Amirkabir University of Technology, 424 Hafez Ave, 15875-4413, Tehran, Iran
E-mail: {hosseini.ec, aakardan}@aut.ac.ir

Abstract

Currently most of the Web-Based learning systems assess the learner's knowledge in terms of the learner's crisp responses to the tests that are taken during the learning process. However, several factors such as successful guesses or choosing a more probable answer lead to uncertainty about the evaluation process. To this end, this paper investigates the use of a novel method based on the Intuitionistic fuzzy set theory to handle inaccurate information about the learner in the assessment process. Also, based on the assessment results, personalized sequence of learning concepts are provided for the learner using Intuitionistic Fuzzy Weighted Averaging (IFWA) operator.

Keywords: E-learning, E-assessment, Personalization, Content sequencing, Intuitionistic fuzzy set theory

Introduction

Rapid Growth of the internet and more specifically the World Wide Web (WWW) provide highly personalized, accessible and interactive sources of information to a widely distributed user base. Hence, Web-based learning has become a new mean to learn such that learners can access learning materials of the applications without time and distance barriers via web services. Web-based learning systems are designed to help learners with diverse knowledge level, skills, backgrounds, preferences and learning styles (Schiaffino et al., 2008).

According to (Dolog et al., 2004) learning in open environments demands even more effective personalization approaches to provide learner orientation and individualized access support. Therefore, Adaptation and personalization is the basic characteristics of most of current web-based learning systems.

Curriculum sequencing or learning path finding is considered as a critical element of personalized learning. It provides each learner the most suitable individually planned sequence of concepts to learn and contents to work with. To provide an effective learning path, most of the personalized tutoring systems consider learner knowledge, backgrounds, and interests for providing adaptive and personalized content sequencing (Brusilovsky, 1996).

However, most of these systems determine the learner knowledge merely through the learner's crisp responses to the tests that are taken during the learning process. There exist some factors that contribute to uncertainty about the evaluation process such as successful guesses or choosing a more probable answer. To illustrate it more, the learner may select a response of the multiple choice question based on the more probable correct answer or similarly the learner may choose an answer by guess. Considering uncertainty in determining the learner's knowledge level was the focus of only few researches. Example of such researches includes the work done in (Chen and Duh, 2008) in which a method is proposed for estimating the learner's ability based on the fuzzy inference mechanism. This method modifies estimation function of the learner's ability according to the learner's answers to two basic questions i.e. the difficulty level and comprehension percentage of each studied courseware. Another work in (Magoulas et al., 2001) evaluates the learner's knowledge level under uncertainty through a Neuro-fuzzy scheme and with regards to the ideas from cognitive science.

This paper investigates the use of a new method based on the Intuitionistic fuzzy set theory to handle inaccurate information about the learner and to provide a personalized sequence of learning material for him/her. The learner has the option of determining the percentgea that he/she believes each answer is right. Then the assessment is done through the Intuitionistic fuzzy set theory. The result of this assessment is later used in sequencing the most suitable learning materials for the learner.

This paper is organized as follows. Section 2 provides a brief review of the Intuitionistic fuzzy sets. Section 3 presents the proposed method for the learner's assessment under uncertainty and describes how this assessment can be used to provide a personalized learning path for the learner. Finally, conclusions are made in Section 4.

Intuitionistic Fuzzy Sets

The theory of Intuitionistic Fuzzy Sets (IFSs) was introduced by Atanassov in 1986 (Atanassov, 1986). It generalizes the fuzzy set theory, and hence all fuzzy sets are IFSs but the converse is not necessarily true. The most significant role of IFSs is in handling inexact and incomplete information.

Besides, it is beneficial in dealing with vague information. According to the predicted results (Atanassov, 2003) and based on the literature researches, IFSs is proved to be useful in various application areas of science and technology. To name the most outstanding areas of applications, IFSs have been successfully applied to medical diagnosis (De et al., 2001) and decision making problems (Xu and Yager, 2008). In the following, we introduce some basic concepts related to IFSs.

Definition 1. Let a set $X = \{x_1, x_2, ..., x_n\}$ be a finite non-empty set, then IFS A in X is defined as [1]:

$$[1] \qquad A = \{< x, t_A(x), f_A(x) \mid x \in X >\}$$

where the functions $t_A : X \to [0,1]$, $f_A : X \to [0,1]$ determines the membership degree and non-membership degree of the $x \in X$ respectively with the condition in [2]:

$$[2] \qquad 0 \le t_A(x) + f_A(x) \le 1$$

All the fuzzy sets can be represented in the form of an IFS as in [3]:

$$[3] \qquad A = \{< x, t_{A'}(x), 1 - t_{A'}(x) \mid x \in X >\}$$

In other words if $t_A(x) + f_A(x) = 1$, then the Intuitionistic fuzzy set is equivalent to the fuzzy set.

Definition 2. Let $a = (t, f)$ be an Intuitionistic fuzzy number. The score function S of a is defined as [4]:

[4] $$S(a) = t - f \qquad S(a) \in [-1,1]$$

Definition 3. Let $a = (t, f)$ be an Intuitionistic fuzzy number, an accuracy function H of a can be represented as [5]:

[5] $$H(a) = t + f \qquad H(a) \in [0,1]$$

According to (Xu, 2007) two Intuitionistic fuzzy numbers can be compared based on the score function and accuracy function:

Definition 4. Let $a_1 = (t_1, f_1)$ and $a_2 = (t_2, f_2)$ be two Intuitionistic fuzzy numbers, $S(a_1) = t_1 - f_1$ and $S(a_2) = t_2 - f_2$ be the score functions of a_1 and a_2, respectively, and let $H(a_1) = t_1 + f_1$ and $H(a_2) = t_2 + f_2$ be the accuracy functions of a_1 and a_2, respectively, then:

If $S(a_1) < S(a_2)$, then a_1 is smaller than a_2, denoted by $a_1 < a_2$;
If $S(a_1) = S(a_2)$, then
(1) If $H(a_1) < H(a_2)$, then a_1 is smaller than a_2, denoted by $a_1 < a_2$;
(2) If $H(a_1) = H(a_2)$, then a_1 and a_2 represent the same information, denoted by $a_1 = a_2$.

Definition 5. Let $a_1 = (t_1, f_1)$ and $a_2 = (t_2, f_2)$ be two Intuitionistic fuzzy numbers, then [6] and [7] are hold:

[6] $$a_1 \oplus a_2 = (t_1 + t_2 - t_1 t_2, f_1 f_2)$$

[7] $$\lambda a_1 = (1 - (1 - t_1)^{\lambda}), (f_1)^{\lambda}), \lambda > 0$$

Definition 6: Let $a_i = (t_i, f_i)\ (i = 1, ..., n)$ be a collection of Intuitionistic fuzzy numbers on X. In this paper, this collection is briefly called Q. Also, Let Intuitionistic Fuzzy Weighted Averaging (IFWA): $Q^n \to Q$, if

[8] $$IFWA_w(a_1, a_2, ..., a_n) = \prod_{i=1}^{n} a_i^{w_i} = (1 - \prod_{i=1}^{n} (1 - t_i)^{w_i}, \prod_{i=1}^{n} (f_i)^{w_i})$$

where $w = (w_{1,} w_2, ..., w_n)^T$ is the weight vector of $a_i (i = 1,2,...,n)$, and $w_i > 0, \sum_{i=1}^{n} w_i = 1$.

Methodology

This section presents the proposed method for evaluating the learner's knowledge level and personalization of the learning path accordingly. The following subsections describe the proposal in detail.

Intuitionistic Fuzzy-Based Learner Assessment

The purpose of this paper is to present a method for assessing the learner's knowledge level under uncertain conditions and while insufficient information is available on the learner's responses to the test items. To this end, IFS theory is applied to evaluate the learner's knowledge level.

A multiple choice question is designed for each of the concepts in the knowledge domain. In the proposed method, the learner has the option to set the correctness percentage for each option of the multiple choice question. This value indicates the percentage that the learner believes each option is the correct answer of the multiple choice question. In case the learner has no idea about the correctness of an option, he/she can leave its correctness percentage blank. The sum of the correctness percentage can be less than or equal to 100. The strength of the IFS theory lies behind this fact, i.e. it enables incorporating the lack of the learner's knowledge about his/her answer.

When the learner sets the correctness percentage for each of the options, the Intuitionistic fuzzy score of the $i-$th tested concept is denoted in the form of [9]:

[9] $$s_i = (t_i, f_i)$$

where s_i presents the Intuitionistic fuzzy score of the $i-$th tested concept which, in this paper, is briefly called IF score. t_i and f_i is the degree of the learner's understanding and not understanding in the $i-$th concept respectively. t_i and f_i can be calculated by [10] and [11] respectively as follows:

[10] $$t_i = \frac{c_i}{100}$$

[11] $$f_i = \frac{w_i}{100}$$

where c_i is the correctness percentage that the learner has assigned to the correct option of the multiple choice question. w_i is the sum of correctness percentage that the learner has assigned to all incorrect options of the multiple choice question.

To illustrate this more, the learner can set the correctness percentage of each test option as follows:

1)10% 2)70% 3)5% 4) –

Suppose that the correct answer of the test is option 2. Based on the values of correctness percentage that are provided by the learner, the value of c_i , and w_i is:

$$c_i = 70, \; w_i = 10 + 5 = 15$$

Now the value of t_i and f_i for the test is calculated according to [10] and [11] respectively.

$$t_i = \frac{70}{100} = 0.7 \qquad f_i = \frac{15}{100} = 0.15$$

Finally, the value of S_i for this test is equal to (0.7, 0.15). The IF score that is obtained using [9] is able to model the uncertain information related to the learner's responses. This value is later used in the content sequencing process for recommending suitable contents to the leaner.

Intuitionistic Fuzzy-Based Content Sequencing

In order to provide a personalized learning path for the learner under uncertain conditions, this paper proposes a method based on the Intuitionistic Fuzzy-Based Learner Assessment. The following steps should be followed for sequencing of the suitable learning contents to learner:

Step 1. The learner selects a learning goal from the list of goals that must be learned throughout the learning process.

Step 2. Since the leaner has no prior learning records in his/her profile, the first ranked list of the learning concepts are presented to the learner according to the instructor's rule. The learner is free to select any concept in the list.

Step 3. The learning content related to the leaner selected concept is then presented to him/her. After the learned finishes studying this content, a test is presented to him/her with the purpose of estimating his/her understanding degree.

Step 4. The learner's response is evaluated based on the IFS theory described in section 3.1.

Step 5. The learner's knowledge level in the tested learning concept is updated in the learner profile according to the value obtained in Step 4.

Step 6. To recommend learning contents based on the learner's feedback, the set of concepts that are related to the Learner-Selected goal will be chosen. Next, the concepts that the learner has already learned will be removed from the set. A concept that the learner has fully learned has the IF score of (1, 0). This means that the learner definitely selected the correct answer. Step 7 is then executed for each unlearned concept in the set.

Step 7. The weighted average of the IF score of the semantically dependent concepts of the selected concept must be determined in this step. This paper considers two kinds of semantic relationships between the selected concept and other concepts of the knowledge domain: 1- Prerequisite Concept that are necessary to perceive the selected concept, and 2- Related Concepts that are related to the selected concept and are in the same Sub-domain. Hence, the value of $IFWA_{total}$ of a concept is calculated as [12]:

$$IFWA_{total} = IFWA_{pre} + IFWA_{related} \qquad [12]$$

where $IFWA_{pre}$ and $IFWA_{related}$ is IFWA of the IF score in Prerequisite and Related Concepts of the selected concept respectively and can be calculated using (8). $IFWA_{total}$ is the total IF score of a concept.

Step 8. The comparison logic for Intuitionistic fuzzy numbers (see Definition 4) is used to rank the concepts based on the $IFWA_{total}$ obtained in Step 7. In this way, the concepts that get higher rank than the other concepts, are the ones that the learner has better IF scores in their Prerequisite and Related Concepts. This will, in turn, help learner better understand the recommended concept, since he/she has better understood its Prerequisite and Related Concept.

Step 9. Step 3 to Step 9 is repeated until the learner learns his/her selected goal.

To illustrate the above steps more, suppose that there are four Concepts A, B, C, D that are related to the learner selected goal and there exists data related to IF score of previous studied concepts. Table 1 shows the IF score of the concepts the learner has studied so far. According to (12) to calculate the value of $IFWA_{total}$ for a typical concept A, $IFWA_{pre}$ and $IFWA_{related}$ must be calculated first. As it is shown in Table 2, Concept A has two Prerequisite Concepts, namely E and F, and no related concept. It should be noted that since A has two Prerequisite Concepts, the weight vector is in the form of $w = (0.5, 0.5)^T$ which assigns equal weight to each of the Prerequisite Concepts according to the expert's view. So, the value of $IFWA_{pre}$, $IFWA_{related}$, and $IFWA_{total}$ can be obtained as follows:

$$IFWA_{pre} = (1-(1-0.60)^{0.5}(1-0.30)^{0.5}, (0.10)^{0.5}(0.60)^{0.5}) = (0.47, 0.25)$$

Since A has no Related Concept, the $IFWA_{related} = (0.0, 0.0)$.

$$IFWA_{total} = IFWA_{pre} + IFWA_{related} = (1-(1-0.47)^{0.5}(1-0.0)^{0.5}, (0.25)^{0.5}(0.0)^{0.5})$$

So: $IFWA_{total} = (0.27, 0.0)$.

The $IFWA_{total}$ can be similarly calculated for the rest of concepts $B, C, D.$ The final results are shown in Table 3.

Table 4. IF score of the studied concepts

Concept	IF score
E	(0.60,0.10)
F	(0.30,0.60)
G	(0.34, 0.20)
H	(1.0, 0.0)

Table 5. Semantic relationships between the concepts

Concept	Prerequisite Concepts	Related Concepts
A	F,E	-
B	-	G
C	E	F
D	H	F

Table 6. Final values of IFWA for each concept

Concept	$IFWA_{total}$
A	(0.27,0.0)
B	(0.19,0.0)
C	(0.27, 0.49)
D	(1.0, 0.0)

According to the data shown in Table 3, concept D is greater that all the other concepts since $S(D) = 1.0 - 0.0 = 1.0$. The final ordered list of concepts that is recommended to the learner is D, A, B, C.

Conclusion

In this paper, the problem of learner's knowledge level assessment by proposing a new method based on the Intuitionistic fuzzy set theory is investigated. To this end, the learner's determines the correctness percentage of each option in the test. The correctness percentage shows denotes that how much the learner is certain with the correctness of each option. This information obtained by the learner response enable considering the impact of successful guesses or choosing a more probable answer. This paper also utilizes the Intuitionistic Fuzzy Weighted Averaging (IFWA) operator to aggregate the Intuitionistic fuzzy information corresponding to each concept. The most suitable sequence of concepts is obtained according to the score function and accuracy function. Future work of this research will explore the impact of other attributes which are influential in determining the suitable sequence of concepts for the learner.

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