

A Unified Interference/Collision Model for Optimal MAC Transmission Power in Adhoc Networks *

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Abstract: *In this paper we address the issue of controlling transmission power in power-aware adhoc networks. We argue that minimum transmission power is not always optimal. Previous work that minimizes the transmission power does not consider both the energy consumed in collision resolution and the energy disbursed to overcome the interference resulting from neighboring nodes. We investigate the basic transmission power control for the 802.11 MAC protocol, in which the control frames and the data frames can be transmitted at different power levels. A unified collision and interference model of a uniformly distributed network is constructed. Based on this model, the end-to-end network throughput and the total energy consumption of the network are examined for different network parameters. For a network with a given node density, our results show the optimal transmission power for control messages and for data messages that will yield maximum throughput and minimum energy consumption per message.*

Keywords: Collision Model; Interference Model; Adhoc Networks; Energy-Efficient MAC.

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1 INTRODUCTION

Adhoc networks have witnessed an explosion of interest in the last few years as they are expected to have a significant impact on the efficiency of many military and civilian applications, such as combat field surveillance, security and disaster management, data gathering, and conferences.

One of the constraints for building an efficient adhoc network is *finite* battery supplies. Since the network nodes are battery operated, and in many cases they are installed in an environment where it may be hard (or undesirable) to retrieve the nodes in order to change or recharge the batteries, it is crucial to design techniques to reduce the node's energy consumption. The nodes need to be energy conserving so that the battery life and hence the total time in which the network is connected and functioning is maximized. Recent research addressed this challenge and various approaches are proposed for each layer of the communication protocol stack [16] to reduce the energy consumption.

Some previous work [9] [18] proposed the idea of minimizing the transmission power and sending the data in a multi-hop fashion to the destination by relaying the packets at intermediate closer nodes. Although the transmission energy is reduced by such scheme, the effect of transmission power control schemes on the total network throughput and the overall energy consumption were not investigated.

Our work is based on the observation that there is a tradeoff in the choice of the transmission power. When reducing the transmission power, the number of nodes included within the transmission range of the sender and competing for wireless channel access is reduced and hence the number of collisions is reduced. However, at every relay node, the data message is relayed and forwarded which increases the probability of collision per message. As a result, in the multihop scheme, collision resolution may end up using more energy than the one hop direct transmission scenario. On the other hand, with respect to interference, it is intuitive that using reduced power minimizes the interference level between neighboring nodes. However, there is an increase in the number of concurrent transmissions because the transmission range of each node is reduced. Consequently, the overall Signal to Interference Ratio (SIR) might degrade when using a lower transmission power.

In this paper, by taking into consideration the energy

wasted in the collision resolutions and the energy used to overcome the interference signal level of neighboring nodes, we argue that the minimum transmission power will not always deliver an optimal energy consumption. We investigate the transmission power adjustment problem to minimize the energy consumption of an adhoc network, based on the 802.11 (CSMA/CA) MAC protocol. A unified collision and interference model is constructed for a uniformly distributed adhoc network. From this model we were able to derive the total network throughput and the total energy consumption in the network.

The rest of the paper is organized as follows: Section 2 presents related work and different power control schemes. Section 3 discusses the background and main assumptions. Sections 4 and 5 describe the interference and the collision models. In Section 6 the average hopcount in the adhoc network is estimated. Section 7 analyzes the total energy consumption in the network. Numerical results are presented in Section 8. We conclude the paper in Section 9.

2 RELATED WORK

Recognizing the challenge of energy consumption in adhoc networks, much research is directed toward energy-efficient protocol design. We can categorize the previous research on power-aware MAC layer into three categories:

Reservation Based Power-Aware MAC tries to avoid collisions in the MAC layer, since collisions may result in retransmissions, leading to unnecessary power consumption. The EC-MAC [26] presented the idea of applying reservation schemes in wireless networks MAC protocols for energy conservation. EC-MAC's definition could be extended to adhoc networks, where a group of nodes may select some type of coordinator to perform the base station functions, as proposed in [2] and [22]. Furthermore, because the coordinator's role consumes the resources of certain nodes, a group of schemes were proposed in which coordinators are rotated among network nodes [11] [12].

Switching off Power-Aware MAC tries to minimize the idle energy consumption by forcing nodes to enter the *doze* mode. For example, PAMAS [25], allows a station to power its radio off but has to keep a separate channel on which the RTS/CTS packets are received. Similarly, Chiasserini [3] allows a station to go to sleep, but a special hardware is required to receive wakeup signals. Also, in [31] the geographical area is partitioned into smaller

*A preliminary version of this work was presented at INFOCOM'04 [7]

grids in each of which only one host needs to remain active to relay packets.

Transmission Power Control came about because the most power is consumed during the transmission mode. much research has been proposed to minimize the transmission power and thus maximize the network lifetime. For example, PARO [9] sends the data to the nearest neighbor in a multihop fashion until reaching the destination. Furthermore, the control frames (RTS/CTS) are sent with maximum power, while the data and acknowledgment frames are sent with reduced power, as will be discussed in Section 3. Other protocols control the transmission power not only based on the distance between the sender and the receiver but also based on different channel conditions. For example, the scheme presented in [23] adjusts the transmission power according to the SNR at the receiver. It allows a node, A, to specify its current transmit power level in the transmitted Request-to-Send (RTS), and allows the receiver node, B, to include a desired transmit power level in the Clear-to-Send (CTS) sent back to A. Analogously, the protocol in [5] chooses an appropriate transmission power based on the packet size.

3 MODEL BACKGROUND

Many previous works have made different assumptions about the radio characteristics of the wireless interface cards, including energy dissipation in transmit, receive, idle and doze modes. Detailed measurement results reported in [4] and [6] emphasized that the most power is consumed in the transmit mode. However, if the transmission/receive durations are small relative to idle time (a typical sensor networks environment), controlling only the *transmission power* rather than putting nodes to sleep may not be the most appropriate way to save energy.

In our work we only analyze the transmission power control schemes because (1) an adhoc network application is different in nature from a sensor network, (2) a considerable portion of the adhoc network lifetime is typically consumed in transmitting and receiving data between nodes, and (3) the most power is consumed in the transmit mode.

In our network model, we assume that a set of homogeneous adhoc nodes are uniformly distributed over a large two dimensional area with a given node density of ρ nodes per unit area. Each node can communicate and receive data directly from all the nodes within its coverage area, where the coverage area of the node is defined by the radius which the control frames can reach

(defined as a_{RTS}). The MAC layer used in such communication is the CSMA/CA protocol with sender-initiated 4-way handshaking scheme (RTS, CTS, DATA, ACK) as defined in the IEEE 802.11 DCF MAC protocol [15]. Based on the uniformly distributed nodes model, all the network hosts will use the same transmission power for DATA/ACK frames and thus will have the same transmission range, defined as a_{data} . Similarly, all hosts use the same power for transmitting the control frames and this has the same coverage area defined by a_{RTS} (which can be different from a_{data}).

Furthermore, we will assume that the time is slotted with slot time τ . We define the *number of time slots* needed to send an RTS packet as L_{RTS} slots. Analogously, the number of time slots needed to send a CTS, a data packet, and an acknowledgment packets are L_{CTS} , L_{data} , and L_{ack} , respectively.

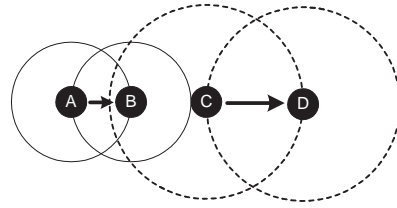


Figure 1: Hidden Terminal Jamming Problem

According to the path-loss radio propagation model, the ratio between the received signal power, P_{Rx} , at distance r from the transmitter, to the transmitted signal power, P_{Tx} , is given by:

$$\frac{P_{Rx}}{P_{Tx}} = C \cdot r^{-\gamma} \quad (1)$$

where C is a constant that depends on the antenna gains, the wavelength, and the antenna heights, r is the transmission distance, and γ is the path loss factor, ranging from 2 (line of sight free space) to 4 (indoor) [17].

As can be observed from Equation (1), to minimize the transmission energy consumption, it is always better to send the data in a multi-hop fashion using relay nodes rather than sending it directly to the destination. A simple power control scheme for the 802.11 RTS/CTS protocol should adjust the transmission energy for data and control frames (RTS/CTS) according to the distance between the sender and the relay node. However, as shown in Figure 1, different power levels among network nodes introduce asymmetric links, a problem known as the ‘‘Hidden Terminal Jamming’’ problem [30]. A hidden node C not sensing an ongoing low power data transmission, can corrupt the data packets being sent from A to B by concurrently transmitting a message to node D. Therefore, as

depicted in Figure 2, the control frames have to be transmitted using a high power level, while the DATA and ACK are transmitted using the minimum power level necessary for the nodes to communicate [8] [23]. As a result, a collision can only occur while transmitting the control frames but not for the DATA/ACK frames.

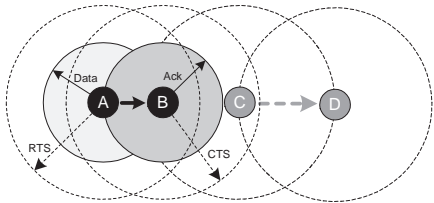


Figure 2: Control Frames with Maximum Power

The expected number of hops, \bar{H} , needed between any source and any destination node is given by:

$$\bar{H} = \lceil \bar{L}/a_{data} \rceil \quad (2)$$

where \bar{L} is the average path length of a message in the adhoc network and a_{data} is the radius by which the DATA/ACK packets are sent, that is, the distance between two consecutive relay nodes. The expected path length, \bar{L} , is a function of the node distribution, dynamic patterns of mobility and traffic patterns in the network [19] [20] [21]. In Section 6 we present a simple way to compute \bar{L} in the adhoc network.

4 INTERFERENCE MODEL

Gupta and Kumar [10] showed that the transmission capacity of an adhoc network is inversely proportional to the square root of the number of nodes in the network due to the increased number of collisions. A collision, as defined by IEEE 802.11, occurs when two or more nodes within the sender coverage area transmits RTS packets at the same time or when an RTS collides with the CTS sent by the receiver node. Collisions can only occur during what is called *Contention Window* [15].

Further, the network throughput is also affected by the interference level caused by hosts concurrently sending their data. Interference occurs during the transmission time of a data frame, where nodes outside the RTS sensing area of the sender and the CTS sensing area of the receiver may concurrently transmit causing a background interference signal that degrades the *Signal to Interference Ratio* (SIR), causing an increase in the *Bit Error Rate* (BER).

The degradation in the total network throughput caused by a low SIR can be a serious problem. We extend the

honey grid model defined in [13], with a new interference model for an adhoc network. We use this model to determine an upper bound on the total injected traffic by each node in the network.

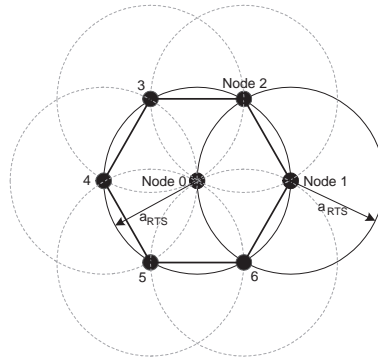


Figure 3: Constellation of Interfering Nodes

Since nodes defer sending any packets upon hearing an RTS/CTS control frame, there will be no source of interference within the node's coverage area. As shown in Figure 3, when Node 0 is transmitting, there will be no interference from any other node within a_{RTS} from it. In the worst case, the first interfering node is just outside the coverage area of Node 0 (e.g., Node 1 at distance $a_{RTS} + \epsilon$ from Node 0). The next interferer could only be outside the coverage areas of both nodes, and in the worst case at the crossing point of two circles each with radius $a_{RTS} + \epsilon$. The constellation of interfering nodes is as shown in Figure 3.

Furthermore, for the worst case scenario of signals interfering with the data packet currently being received at Node 0 there are at most 6 interfering nodes at distance $a_{RTS} + \epsilon$, and on the next interfering ring, at distance $2 \cdot (a_{RTS} + \epsilon)$, there are at most 12 interfering nodes and so on. This results in the *Honey Grid Model*, depicted in Figure 4.

However, not all the interfering nodes can concurrently transmit their data frames as shown in Figure 5. Let Node R (within Node 0's coverage area) wants to communicate with Node 0. Node R initiates the communication by sending an RTS, Node 0 responds with a CTS, and all nodes with the coverage area (defined by a_{RTS}) of Node R should defer their transmission. As shown in Figure 5(a) the coverage area of Node R may include two interferers from the first interfering ring, causing them to withhold their transmissions and not causing any interfering signal to Node 0. In the worst case interference scenario, only one interferer is included in the coverage area of Node R, as shown in Figure 5(b). With similar reason-

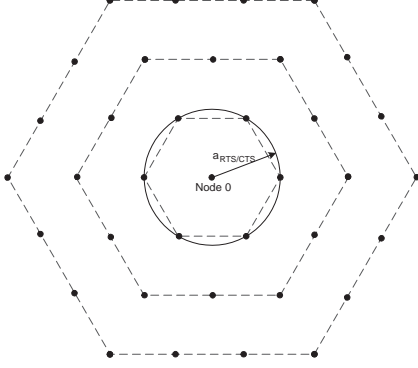


Figure 4: Honey Grid Interference Model

ing, we can argue that each of the other 5 interferers (in first ring) is communicating with a host in the interferer's coverage area and when this host replies with a CTS, this host shuts down, in the worst case, only one other interferer. Hence, there can be at most 3 interferers at the first ring, 6 at the second ring and $3i$ nodes at the interference ring i .

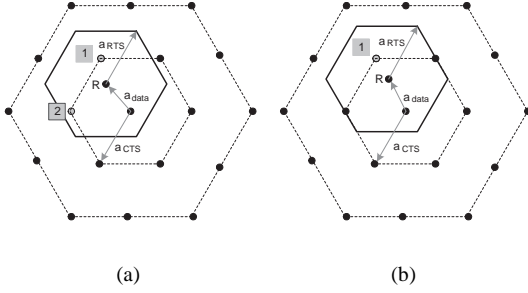


Figure 5: Interfering Nodes per Ring

Assume that the "own" traffic originated from each node is μ messages per second, and on average there are $(\bar{H} - 1)$ relay nodes between any source and destination pair. Then, the expected volume of relay traffic reaching any node is given by $\mu \cdot (\bar{H} - 1)$. Consequently, the total traffic per node can be given:

$$\begin{aligned} \text{total traffic per node} &= \text{own traffic} + \text{relay traffic} \\ &= \mu + \mu \cdot (\bar{H} - 1) = \mu \cdot \bar{H} \end{aligned} \quad (3)$$

In order to get an upper bound on the own traffic produced by each node and injected into the network, μ , we compute the worst case interference scenario, which occurs when all the interferers are actively transmitting. We add the received interference power from 3 nodes in the first ring at distance a_{RTS} , and 6 nodes in the second ring

at $2a_{RTS}$, and so on. Since the network is uniformly distributed, we can assume that all the data/ack packets are sent with signal level P_{data} covering a radius of a_{data} . On the other hand, the control frames are sent with a high power covering a radius of a_{RTS} . From Equation (1), for a fixed Bit Error Rate, the ratio between the control packets transmission power to the data packets transmission power is equal to the ratio of distances raised to the power of γ . Hence, the power by which the control frames are sent, $P_{RTS/CTS}$, is given as:

$$P_{RTS/CTS} = P_{data} \cdot \left(\frac{a_{RTS}}{a_{data}} \right)^\gamma \quad (4)$$

where γ is the path loss factor (see Equation (1)).

Let $T_{total} = L_{RTS} + L_{CTS} + L_{data} + L_{ack}$ be the total time to send one frame (without any retransmissions). Then the average interference level, I_r , of a single interferer located at distance r from the receiving node is

$$\begin{aligned} I_r &= q \cdot (P_{data} \cdot r^{-\gamma} \cdot \frac{L_{data} + L_{ack}}{T_{total}} \\ &+ P_{data} \cdot \left(\frac{a_{RTS}}{a_{data}} \right)^\gamma \cdot r^{-\gamma} \cdot \frac{L_{RTS} + L_{CTS}}{T_{total}}) \end{aligned} \quad (5)$$

where q is the probability of transmission per node. The first term inside the brackets represents the interference level caused by the data/ack packets with power P_{data} , and the second term accounts for sending the control frames (RTS/CTS) with the power defined in Equation (4).

Using Equation (5), we can compute the total interference at Node 0 caused by other network nodes in the honey grid model as:

$$\begin{aligned} I &= \frac{3 \cdot q \cdot P_{data} \cdot a_{RTS}^{-\gamma}}{T_{total}} \sum_{i=1}^{\infty} \{ i^{-(\gamma-1)} \times [(L_{data} + L_{ack}) \\ &+ \left(\frac{a_{RTS}}{a_{data}} \right)^\gamma (L_{RTS} + L_{CTS})] \} \end{aligned} \quad (6)$$

This is done by substituting distance r with $i \cdot a_{RTS}$ (the radius of the i^{th} interfering ring) and summing up for all $3i$ interfering nodes in this ring. Since the series in Equation (6) is a converging series, the interference level caused by a distant node can be neglected if it is below a certain threshold, which depends on the type of the interface card used.

The SIR at Node 0 can be derived as the ratio between the signal level of the sender at distance a_{data} away from Node 0 to the total interference level at this node, as defined by Equation (6). Hence, the SIR can be given as:

$$SIR = G \cdot \frac{P_{data} \cdot a_{data}^{-\gamma}}{I} \quad (7)$$

where G is the spread spectrum ‘‘Processing Gain’’ [24] used in the network physical layer.

Assuming that the total traffic per node is a Poisson process then the probability that a node transmits, q , is given as:

$$q = 1 - e^{-\mu \cdot \bar{H}} \quad (8)$$

By using the value of \bar{H} as given in Equation (2) and by substituting q in Equation (6) and then substituting back the total interference level, I , in Equation (7), we can calculate the maximum traffic that a node can produce, μ , while keeping $SIR = SIR_{min}$ at all other nodes:

$$\mu = -\frac{a_{data}}{\bar{L}} \cdot \ln \left[1 - \frac{T_{total} \cdot G \cdot a_{data}^{-\gamma}}{3 \cdot SIR_{min} \cdot a_{RTS}^{-\gamma} \cdot \sum_{i=1}^{\infty} i^{-(\gamma-1)}} \cdot \frac{1}{(L_{data} + L_{ack}) + (a_{RTS}/a_{data})^\gamma \cdot (L_{RTS} + L_{CTS})} \right] \quad (9)$$

As illustrated in Section 8, μ will be used to derive and evaluate the total network throughput. The network throughput is defined as the sum of the throughputs of each node that can concurrently transmit without causing a collision. Evaluating the total throughput at different values for both a_{data} and a_{RTS} will demonstrate the presence of a certain optimum transmission range for the control and data messages at which the throughput is maximized.

5 COLLISION MODEL

The nodes included within the coverage area of a certain host may send control messages that collide with the RTS/CTS frames transmitted by this node. A collision resolution scheme (exponential backoff) [14] is applied whenever a collision is detected. The higher the number of collisions, the lower the network throughput and the higher the energy consumed resolving these collisions. We modify and apply the collision model proposed in [29] for a uniformly distributed multihop adhoc network, and using this model, we derive the effect of collisions on both the throughput and the total energy consumption.

The wireless channel state transition diagram around a certain node x is shown in Figure 6. *IDLE* is the state when channel around node x is sensed idle, and its duration is for one time slot, τ . The *Transmit* state indicates that a successful four-way handshake is completed, and hence, its duration is $T_{transmit} = L_{RTS} + L_{CTS} + L_{data} + L_{ack}$. The *RTS-col* state indicates that multiple hosts within the coverage area of node x transmit RTS frames concurrently, causing an RTS collision; its duration is $T_r = L_{RTS}$. Finally, the *CTS-col* state indicates that a terminal hidden from node x sends some packets that collide

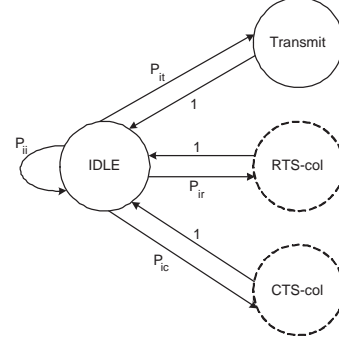


Figure 6: Wireless Channel State Transition Diagram

at the receiver with the RTS being received or the CTS being sent; its duration is $T_c = L_{RTS} + L_{CTS}$.

In our analysis, we assume that the size of the *Contention Window* (CW) is held constant. As proved in [14] and [1], the probability that a fully saturated node, a node that is always having a packet waiting in the output buffer to be sent, transmits at a given time slot, p , is given by

$$p = \frac{2}{CW + 1} \quad (10)$$

Using p we can derive the transition probabilities for the collision model as follows. The probability P_{ii} is the transition probability from *IDLE* to *IDLE*, that is, the probability that none of the nodes within the coverage area of x transmits at this time slot. P_{ii} is given by:

$$P_{ii} = (1 - p)^M \quad (11)$$

where $M = \rho \cdot \pi a_{RTS}^2$ is the total number of nodes included in the coverage area of node x .

The probability P_{it} is the transition probability from *IDLE* to *Transmit*. It is the probability that exactly one node transmits at this time slot and starts a successful four-way handshake (i.e., other nodes withhold their transmission). P_{it} is given by:

$$P_{it} = M \cdot \Pi_s \cdot (1 - p)^{M-1} \quad (12)$$

where Π_s denotes the probability that a node begins a successful four-way handshake at this time slot. Π_s is a function of the number of hidden terminals and the distance between the sender and the receiver as will be discussed later in this section.

The probability P_{ir} is the transition probability from *IDLE* to *RTS-col*. It is the probability that more than one node transmits an RTS packet at the same time slot. In other words, P_{ir} is

(1 - probability that none of the nodes transmits - probability that exactly one node transmits):

$$P_{ir} = 1 - (1 - p)^M - M \cdot p \cdot (1 - p)^{M-1} \quad (13)$$

Finally, P_{ic} , the transition probability from *IDLE* to *CTS-col*, can be simply computed as:

$$P_{ic} = 1 - P_{ii} - P_{it} - P_{ir} \quad (14)$$

Having calculated P_{ii} , P_{it} , P_{ir} and P_{ic} , the equilibrium equations of the wireless channel state transition diagram can be deduced and solved, so that the *Transmit* state limiting probability, θ_t , can be computed. θ_t represents the percentage of time in which the node is successfully transmitting, or in other words, it is the ratio between successful transmission time to the total network time (defined as the summation of transmission time and contention time). The solution of the state model equilibrium equations is:

$$\theta_t = \frac{P_{it}}{1 + P_{it} \cdot T_{transmit} + P_{ir} \cdot T_r + P_{ic} \cdot T_c} \quad (15)$$

All the terms of Equation (15) have been derived with the exception of P_{it} as it depends on Π_s , the probability that a node starts a successful four-way handshake in the given time slot. In order to determine, Π_s , the state transition diagram of a wireless node is constructed as shown in Figure 7. Node x is in the *succeed* state when it can complete a successful four-handshake with the other nodes, it enters the *fail* state when the node initiates an unsuccessful handshake, and the *wait* state accounts for deferring for other nodes. Π_s is the limiting probability of the *succeed* state, as computed next.

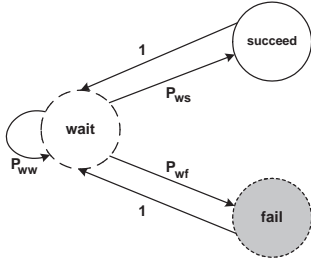


Figure 7: Wireless Node State Transition Diagram

We define $B(a_{data})$ to be the hidden area from node x when communicating with node R located at a_{data} away from it, as illustrated in Figure 8. Takagi [27] has proved that $B(a_{data})$ takes the form:

$$B(a_{data}) = \pi \cdot a_{RTS}^2 - 2 \cdot a_{RTS}^2 \cdot \left\{ \arccos\left(\frac{a_{data}}{2 \cdot a_{RTS}}\right) - \frac{a_{data}}{2 \cdot a_{RTS}} \cdot \sqrt{1 - \frac{a_{data}^2}{4 \cdot a_{RTS}^2}} \right\} \quad (16)$$

The number of nodes hidden from the sender, computed as $p \cdot B(a_{data})$, are not included in the sender coverage area but are within the receiver node coverage *and* can collide with the RTS frame being received or the CTS frame transmitted by the receiver.

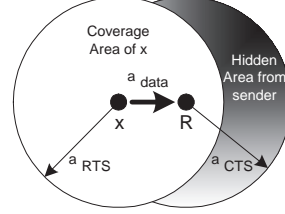


Figure 8: Hidden Area From the Sender

The transition probability P_{ww} , from *wait* state to *wait* state, is the probability that neither node x nor any node within its coverage area is initiating any transmissions. P_{ww} is given by:

$$P_{ww} = (1 - p)^M \quad (17)$$

The transition probability, P_{ws} , from *wait* state to *succeed* state is the probability that node x transmits at this time slot and none of the terminals within a_{RTS} of it transmits in the same slot, and also that none of the hidden nodes in $B(a_{data})$ transmits for $(L_{RTS} + L_{CTS})$ slots. P_{ws} can be written as:

$$P_{ws} = p \cdot (1 - p)^M \cdot [(1 - p)^{p \cdot B(a_{data})}]^{L_{RTS} + L_{CTS}} \quad (18)$$

Finally, the transition probability P_{wf} , from *wait* state to *fail* state can be simply calculated as:

$$P_{wf} = 1 - P_{ww} - P_{ws} \quad (19)$$

Solving the equilibrium equations of the wireless node state transition diagram, the limiting probability of state *succeed*, Π_s can be given by:

$$\begin{aligned} \Pi_s &= \frac{P_{ws}}{2 - P_{ww}} \\ &= \frac{p \cdot (1 - p)^M \cdot [(1 - p)^{p \cdot B(a_{data})}]^{L_{RTS} + L_{CTS}}}{2 - (1 - p)^M} \end{aligned} \quad (20)$$

The value of Π_s is substituted into Equation (12). Then the obtained value of P_{it} is substituted back into Equation (15) so that θ_t , the ratio between successful transmission time to the total network time, can be derived. As illustrated in Section 8, the value of θ_t will be used to evaluate the total network throughput. Also, θ_t will be used to get the percentage of the total time consumed in collisions, hence, the energy consumption can be evaluated.

6 ESTIMATION OF AVERAGE HOP COUNT

As mentioned in Section 3, the expected path length is a function in the node distribution and the dynamic traffic patterns in the network. In this section we present a simple way to compute the average hopcount (\bar{H}) when having different types of traffic for uniformly distributed stationary nodes.

6.1 Random Traffic Pattern

In the random traffic pattern, the source and the destination nodes of each traffic flow are randomly chosen from the network nodes.

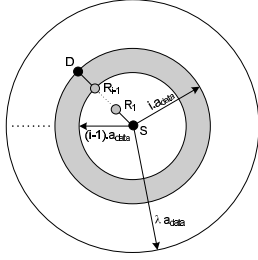


Figure 9: A Route of Length i Hops

We assume, as an approximation, that the sender node is at the network center and away from the boundaries. As a result, and as depicted in Figure 9, the probability of having a route of length i hops from the sender (S) to the destination (D) is proportional to the number of relay nodes (R_j) included in the area inscribed by two discs of radii $i \cdot a_{data}$ and $(i-1) \cdot a_{data}$ (shaded area in Figure 9), and is given by:

$$p(H = i) = \frac{\rho \cdot \pi \cdot ((i \cdot a_{data})^2 - ((i-1) \cdot a_{data})^2)}{N} \quad (21)$$

where N is the total number of nodes in the network and ρ is the node density. If the total radius of the network is denoted by $\lambda \cdot a_{data}$ then

$$\lambda = \sqrt{\frac{N}{\rho \cdot \pi \cdot a_{data}^2}} \quad (22)$$

and thus $p(H = i)$ can be evaluated as:

$$p(H = i) = \frac{2 \cdot i - 1}{\lambda^2} \quad (23)$$

As a result, the expected hopcount \bar{H} can be computed as:

$$\bar{H} = \sum_{i=1}^{\lambda} p(H = i) \cdot i = \frac{2 \cdot (\lambda + 1)^3}{3 \cdot \lambda^2} - \frac{3 \cdot (\lambda + 1)^2}{2 \cdot \lambda^2} + \frac{5 \cdot (\lambda + 1)}{6 \cdot \lambda^2} + \frac{5}{6 \cdot \lambda^2} \quad (24)$$

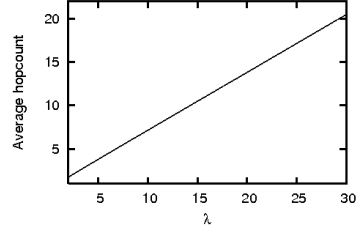


Figure 10: Average Hopcount in Random Traffic

As shown in Figure 10, the average hopcount for the random traffic pattern is almost linearly increasing with the increase in the total network radius.

6.2 Local Traffic Pattern

Li et al. [19] noticed that some networks (e.g., LAN users) may have a predominantly local traffic pattern in which it is more probable that a node communicates with a near host rather than a farther one. The traffic pattern in that case can be described as a Pareto Law (also known as power-law distribution), as given by Equation 25:

$$p[L > x] \propto x^{-k} \quad (25)$$

where $p[L > x]$ is the probability that the path length is larger than x and is proportional to an inverse power of x , where k is a positive constant that represents the “locality” of traffic. The larger the value of k is, the closer the destinations are to the sources. It should be noted that L is lower bounded by a value ε that is a function in the node density (ρ). ε is determined such that there is at least one receiver in the transmission range of the sender, hence, $\varepsilon = \sqrt{2/\rho \cdot \pi}$.

Similar to the random traffic pattern case, the expected hopcount \bar{H} can be computed as:

$$\bar{H} = \sum_{i=1}^{\lambda} p(H = i) \cdot i = \frac{\int_{\varepsilon/a_{data}}^1 x^{-(k+1)} dx}{\int_{\varepsilon/a_{data}}^{\lambda} t^{-(k+1)} dt} + \sum_{i=2}^{\lambda} \frac{i \cdot \int_{x=i-1}^i x^{-(k+1)} dx}{\int_{\varepsilon/a_{data}}^{\lambda} t^{-(k+1)} dt} \quad (26)$$

Using Equation (26) the average hopcount in the network can be computed for the local traffic pattern. Table 1 presents an instance of such computation for two networks at different values for the locality index k . For comparison the \bar{H}_{random} values for the two networks are 20.49 and 10.49 respectively.

Table 1: Average Hopcount

	Network 1 ($\rho = 1, \lambda = 30$)	Network 2 ($\rho = 3, \lambda = 15$)
k=0	8.745	4.752
k=1	3.455	2.212
k=2	2.061	1.322

7 ENERGY COMPUTATION

In addition to transmitting the RTS/CTS packets with high transmit power and the data packets with reduced power, transmission energy is also consumed in retransmitting control frames in case of collisions. We first investigate the power consumption in data and control message transmissions. Second, we derive the time spent in successful transmission and that consumed during collisions. The total energy is the product of the power consumed and the time spent in transmissions and collisions.

Power Consumption: Due to the free space power loss, as indicated by Equation (1), the transmission power for data messages, P_{data} , is:

$$P_{data} = C \cdot a_{data}^{\gamma} \quad (27)$$

where C is a constant that depends on the wireless network interface card and γ is the path loss factor.

Similar to the data frames, the power consumed in transmitting the RTS control frames is also proportional the transmission distance (a_{RTS}) raised to the power of γ . However, retransmissions occur due to collisions with the RTS frames sent by other nodes. Hence, the power consumption in RTS transmission, P_{RTS} , is given by:

$$P_{RTS} = \sum_{i=1}^M \binom{M}{i} \cdot i \cdot C \cdot a_{RTS}^{\gamma} \cdot p^i \cdot (1-p)^{M-i} \quad (28)$$

where p is the probability that a node transmits at this time slot as given by Equation (10). P_{RTS} is the summation of the power consumed in sending i RTS frames multiplied by the probability that i nodes transmit an RTS frame at the same time slot, where i ranges from 1 to M and M is the total number of nodes included in the sender coverage area.

Furthermore, P_{CTS} , the power consumed in transmitting the CTS frame, takes the same form as P_{RTS} . However, the number of nodes contending for accessing the wireless channel are those nodes hidden from the sender as illustrated by Figure 8. The number of hidden terminals, K , can be given as $\rho \cdot B(a_{data})$. Hence, P_{CTS} takes the form:

$$P_{CTS} = \sum_{i=1}^K \binom{K}{i} \cdot i \cdot C \cdot a_{RTS}^{\gamma} \cdot p^i \cdot (1-p)^{K-i} \quad (29)$$

Time: By definition, θ_t in Equation (15) is the percentage of time the node is in a successful data transmission state to the total consumed time (the summation of transmission time and contention time). Hence the total consumed time, T_{total} , can be given as:

$$T_{total} = \frac{T_{transmit}}{\theta_t} = \frac{L_{RTS} + L_{CTS} + L_{data} + L_{ack}}{\theta_t} \quad (30)$$

Solving the equilibrium equations of the wireless channel state transition diagram, discussed in Section 5, we can derive the percentage of time the system is in *RTS-col* relative to the total time, θ_r , as:

$$\theta_r = \frac{\theta_t}{P_{it}} \cdot P_{ir} \quad (31)$$

where P_{it} and P_{ir} are given by Equations (12) and (13) respectively. Similarly, the percentage of time the system is in *CTS-col* relative to the total time, θ_c , is:

$$\theta_c = \frac{\theta_t}{P_{it}} \cdot P_{ic} \quad (32)$$

Hence the total contention time during collisions and control frame retransmissions has an RTS component, $T_{RTS} = \theta_r \cdot T_{total}$, and a CTS component, $T_{CTS} = \theta_c \cdot T_{total}$.

Energy: Having derived both the time and power consumption in transmitting the data frames and in the collision/retransmissions, we can simply evaluate the total expected energy consumption in the network, E , by multiplying the energy per hop by the expected number of hops, \bar{L}/a_{data} , in the network:

$$E = \frac{\bar{L}}{a_{data}} \cdot \{P_{data} \cdot T_{transmit} + P_{RTS} \cdot T_{RTS} + P_{CTS} \cdot T_{CTS}\} \quad (33)$$

As discussed in Section 8, using Equation (33) we can evaluate the total energy consumption in the network and also investigate the energy consumption per message for different node transmission power ranges, and, thus, we determine the optimum transmission power for both the control and data messages based on the given network parameters.

8 NUMERICAL RESULTS

Using the analytical equations previously derived and substituting the different network parameters by the values shown in Table 2, we present results for the network throughput and the total energy consumption for a uniformly distributed adhoc network.

Table 2: Network Parameters

Parameter	Symbol	Value
RTS packet time	L_{RTS}	13 slot time
CTS packet time	L_{CTS}	12 slot time
Data packet time	L_{data}	287 slot time
Ack packet time	L_{ack}	12 slot time
Processing gain	G	10 db
SIR Threshold	SIR_{min}	21 db
Path loss factor	γ	2
Node density	ρ	[1, 3] node/ d^2
Contention window	CW	[16, 1024] slot time
Expected path length	\bar{L}	16 d

The first five parameters are derived from the IEEE 802.11 specifications [15]. SIR_{min} is set according to [28] for 10% Packet Error Rate (PER). γ is set to 2 for the free space line of sight case. ρ and CW are simulation parameters that are changed to investigate their effect on the network throughput and energy consumption; CW ranges from $CW_{min} = 16$ to $CW_{max} = 1024$ slot time [1]. Moreover, the unit of distance is taken to be an arbitrary unit of length d , in which the expected path length, the data transmission range (a_{data}), and the control frame transmission range (a_{RTS}) are given.

As shown in Section 6, the average hopcount \bar{H} for the random traffic pattern is linearly increasing with the total network radius λ , thus according to Equation (22), \bar{H} is also linearly increasing with $1/a_{data}$. As a result, for random traffic pattern and according to Equation (2), the average path length \bar{L} can be assumed to be constant. In our experiments \bar{L} is set to 16 (changing \bar{L} will only have a linear effect on the results). Later on in this section we investigate the case when we have local traffic pattern in which the assumption of constant \bar{L} is not valid.

If we assume that the network is partitioned into several flows, where a flow is each node that can transmit at the same time without causing a collision, then the total network throughput can be defined as the sum of throughputs of each flow. We define σ to denote the number of nodes that can concurrently transmit at the same time without causing a collision divided by the total number of network nodes. As discussed in Section 4, σ can be defined as the total number of nodes in each interfering ring divided by

the total number of network nodes. Hence, for a large network of radius λ , σ can be given as:

$$\sigma = \frac{1}{\rho \cdot \pi \cdot \lambda^2} \cdot \sum_{i=1}^{\frac{\lambda}{a_{RTS}}} 3 \cdot i \quad (34)$$

$$\approx \frac{3}{2 \cdot \rho \cdot \pi \cdot a_{RTS}^2}$$

where ρ is the node density and the number of interference rings in the network is given by λ/a_{RTS} .

Let μ be the traffic produced by each node in the network, expressed in messages/second. Thus, the total throughput per node can be simply obtained as the product of the average number of concurrently transmitting nodes, the ‘‘own’’ produced traffic per node, and the percentage of time the node is actually in a successful transmission status.

$$\text{Total Throughput per node} = \sigma \times \mu \times \theta_i \quad (35)$$

It should be mentioned that, the units of the results (throughput and energy) in this section are irrelevant since we are only interested in the shape of the curves, and also since the units depend on the choice of the distance unit value d .

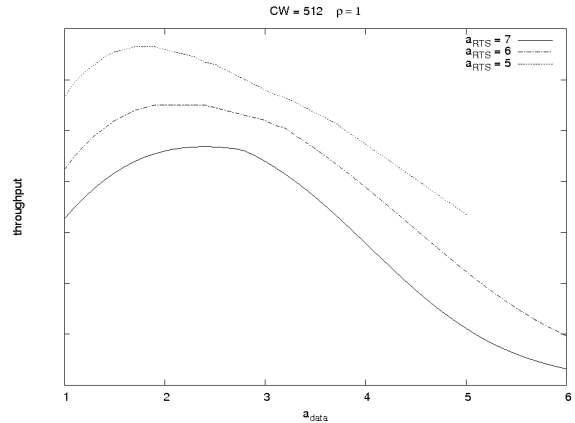


Figure 11: Total Network Throughput per Node

Figure 11 shows the results for the network throughput per node. These results emphasize the fact that for a given a_{RTS} there is an optimal distance (a_{data}), by which the data packets should be sent in order to maximize the network throughput. It should be noted that, $a_{data} \leq a_{RTS}$ because the control frames are sent with a high power to prevent the ‘‘Hidden Terminal Jamming Problem’’, as previously mentioned. The lower bound on a_{data} is a function

of ρ and determined such that there is at least one receiver in the transmission range of the sender.

As shown in Figure 11, at small a_{data} the node is sending to a near neighbor, which increases the number of hops needed per message reducing the network throughput. As a_{data} increases, the number of hops per message decreases and the throughput increases. For a given a_{RTS} the maximum throughput is up to 30% higher than the throughput at the minimum value for a_{data} ; this proves that it is not always optimal to use the minimum value for a_{data} as proposed in previous work [8] [23]. As a_{data} increases more, the network throughput drops because the number of hidden terminals increases, leading to an increase in the number of collisions.

On the other hand, the total network throughput degrades as a_{RTS} increases. Increasing the a_{RTS} reduces the interference level since more nodes defer their transmission when the data frame is being transmitted. But this effect seems to be overwhelmed by the collision effect as the number of colliding nodes trying to access the medium increases, causing an increased number of collisions of control messages and thus reduced throughput.

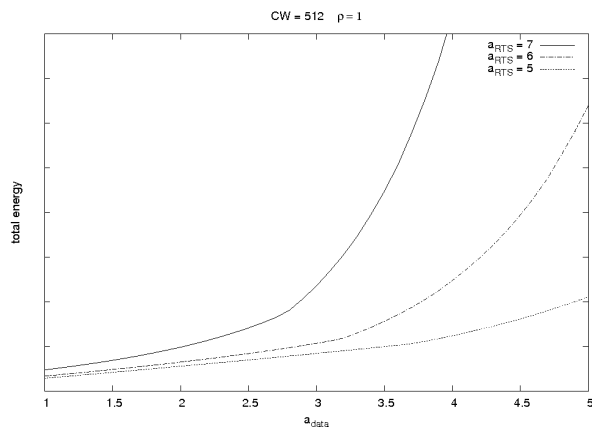


Figure 12: Total Energy Consumption

Figure 12 shows the results for the total network energy consumption. As a_{data} increases, the energy consumed in data messages transmission dominates the total energy consumption. At large a_{data} the number of hidden terminals from the sender increases and the energy wasted during CTS collision dominates the network energy consumption. Additionally, the message reaches its destination with fewer hops, but the energy per hop is high due to the r^γ factor in Equation (27).

By evaluating the energy consumption per message (that is, the energy normalized by the throughput) in the network, an interesting result is obtained. As shown in

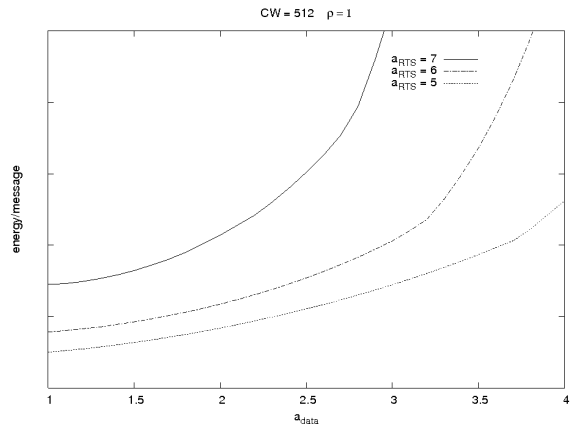


Figure 13: Total Energy Consumption per Message

Figure 13 the energy consumption per message increases with larger a_{RTS} . However, the effect of a_{data} is much less pronounced, leading to the choice of a slightly larger a_{data} than the minimum, at the benefit of increasing throughput (see Figure 11).

The results from Figures 11–13 show that, for a uniform network, the power by which the control frames are transmitted should be minimized to the level that just keeps the network fully connected. Further, a_{data} should not be necessarily set to the smallest possible value.

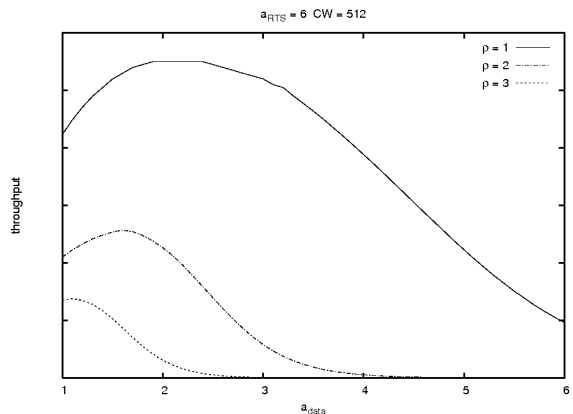


Figure 14: ρ Effect on Throughput per Node

Figure 14 shows the effect of changing the node density on the network throughput. As expected, when the density (number of nodes) increases the throughput decreases since the number of collisions increase as more nodes are contending to access the wireless channel. However, the reduction in the throughput (e.g., the large drop between $\rho = 1$ and $\rho = 2$) is much larger than that reported by [10]

since we take into account the combined effect of both the collision and interference.

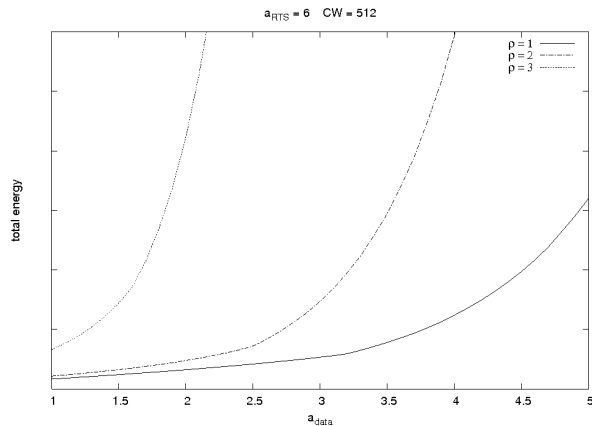


Figure 15: ρ Effect on Energy Consumption

The effect of changing the node density on the overall energy consumption is shown in Figure 15. At a specific hop length (a_{data}) the number of nodes within the node coverage area increases with the increase of ρ and hence the number of contending nodes to access the wireless channel increases leading to an increase in the energy wasted during collision and retransmissions.

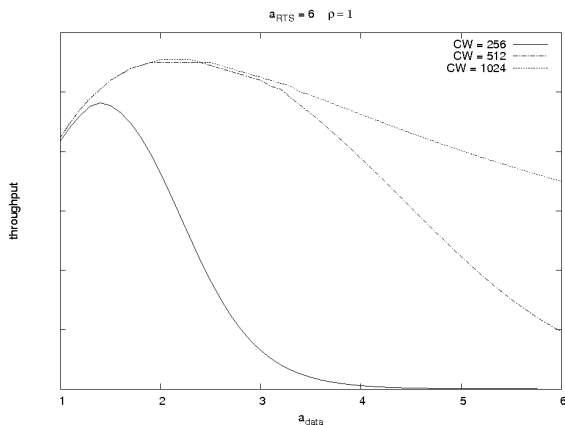


Figure 16: CW Effect on Throughput per Node

Figure 16 shows the effect of changing the contention window size on the network throughput. From Equation (10), with smaller CW the probability that a node transmits at the current slot time increases and hence the probability of collision increases. Thus, the smaller the CW, the lower the throughput. It should also be noted that as CW decreases the optimal a_{data} approaches its minimum value. Therefore, at smaller contention window size,

it is better to use the minimum data power between relay nodes.

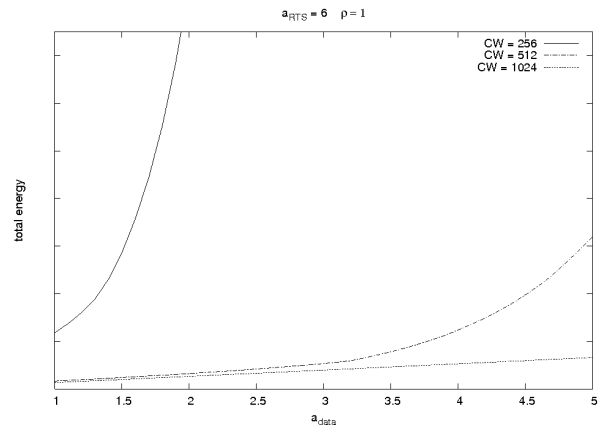


Figure 17: CW Effect on Energy Consumption

The effect of changing the contention window size on the energy consumption is shown in Figure 17. When CW decreases, the probability that a node transmits at the current slot time increases and hence the probability of collision increases, causing more energy to be wasted during collision.

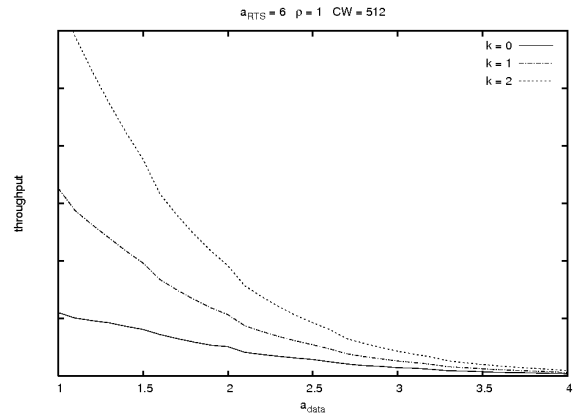


Figure 18: Locality Index Effect on Throughput

All the previous results are obtained under the assumption of random traffic pattern and, hence, the assumption of a fixed \bar{L} holds. For the local traffic pattern case, this assumption is not valid anymore. Therefore, the value of \bar{L}/a_{data} in Equation (9) and in Equation (33) has to be replaced with the value of \bar{H} in the local traffic pattern as defined by Equation (26).

Figures 18 and 19 show the effect of changing the traffic locality index on the network throughput and the en-

ergy consumption respectively. When we have a network of 2000 nodes, for local traffic pattern, it is always optimal for both the network throughput and total energy consumption to use the minimum a_{data} which is equivalent to using the minimum transmission power for data and ACK frames. Moreover, as indicated in Figures 18 and 19, the more local the traffic is (higher value for k), the higher the network throughput is and lower energy is consumed to deliver the packets to their final destinations.

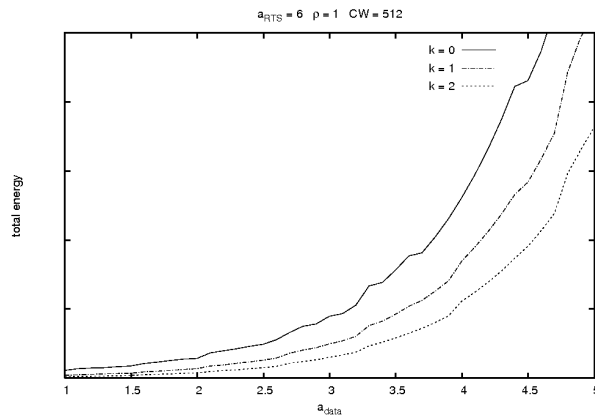


Figure 19: Locality Index Effect on Energy Consumption

9 CONCLUSION

In our work we investigated the effect of transmission power control for power-aware adhoc networks on the overall throughput and energy savings in the network. We have shown that it is *not* always optimal to send the data packets to the nearest neighbor. For a given expected path length and a given node density, we derived expressions to compute the optimal transmission distance that will yield maximum throughput of the network and minimized energy consumption per message.

Furthermore, we showed that the control messages should not be sent with the maximum power as was suggested by previous work. By investigating the energy consumption per message, we were able to show that the transmission power for control frames should be minimized to the extent of keeping the network connected.

Lastly, our work suggests that the contention window should be initialized to a larger value than currently suggested by protocol specifications.

This work can be extended in several ways. First, the idle energy consumption in the network and the energy consumed in the relay nodes during receiving the traffic

should be investigated in addition to transmission energy. Second, the delays in the network should be accounted for when setting the transmission power for control and data frames. Third, studying the effect of changing the selection criteria of relay nodes on network lifetime is critical. The relay nodes may be selected based on different factors, such as their current battery capacity, in addition to their distance from the sender and the receiver.

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