

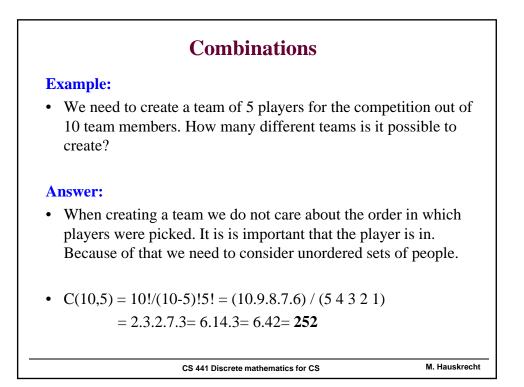
Combinations

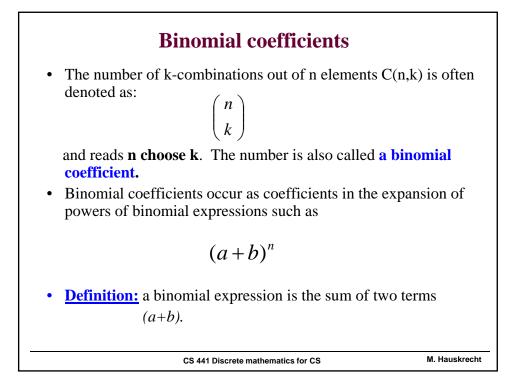
Theorem: The number of *k*-combinations of a set with *n* distinct elements, where *n* is a positive integer and *k* is an integer with $0 \le k \le n$ is

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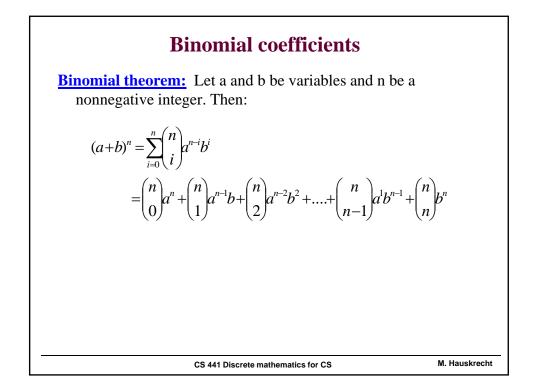
$$C(n,k) = \frac{n!}{(n-k)! \, k!}$$

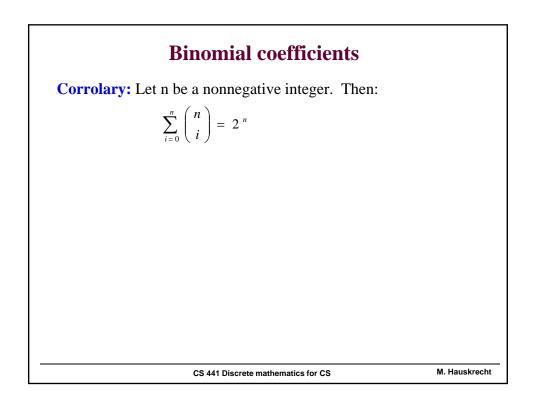
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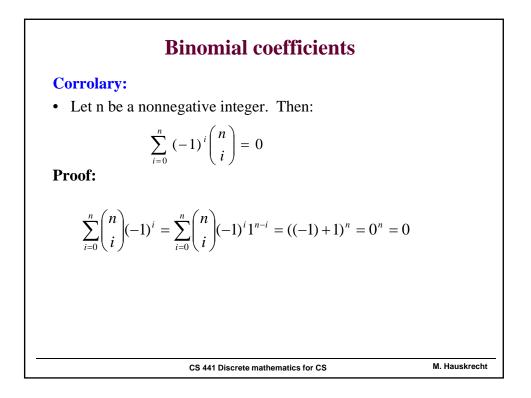


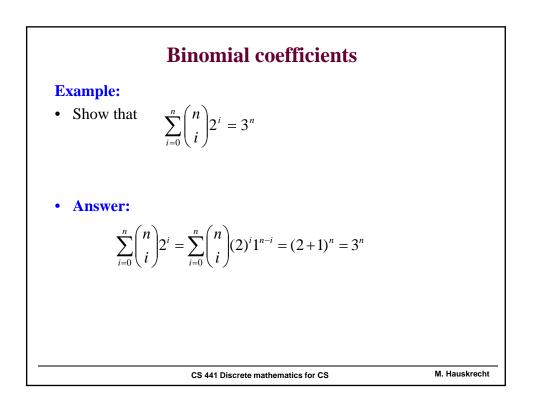


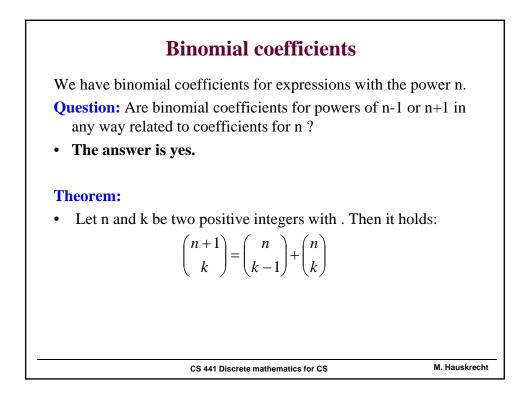
Binomial coefficients
 Example: Expansion of the binomial expression (a+b)³.
$(a+b)^{3} =$ $(a+b)(a+b)(a+b) =$ $(a^{2}+2ab+b^{2})(a+b) =$ $a^{3}+2a^{2}b+ab^{2}+a^{2}b+2ab^{2}+b^{3} =$
$1a^{3} + 3a^{2}b + 3ab^{2} + 1b^{3}$ $1 3 3 1 \longleftarrow \text{Binomial coefficients}$ $\begin{pmatrix} 3 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \end{pmatrix}$
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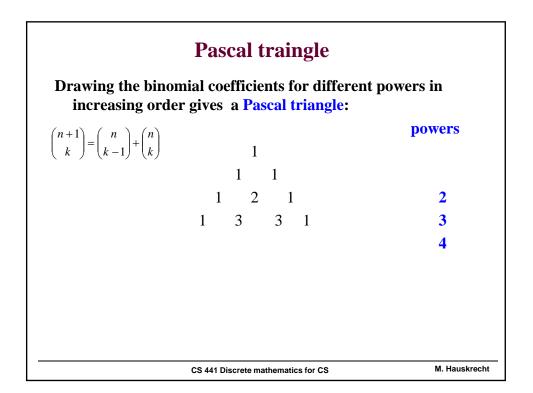


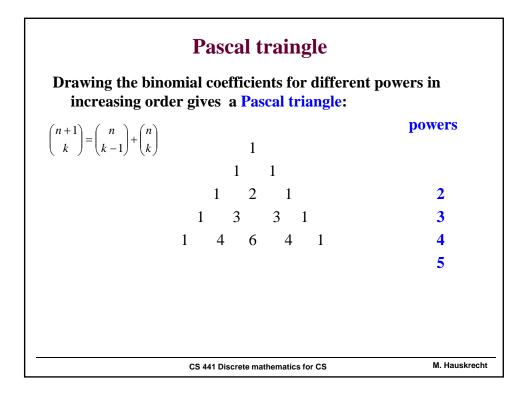


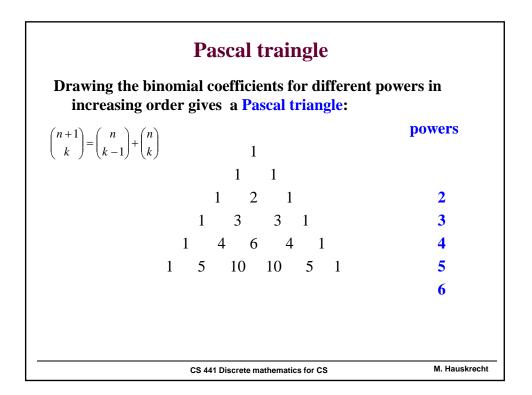


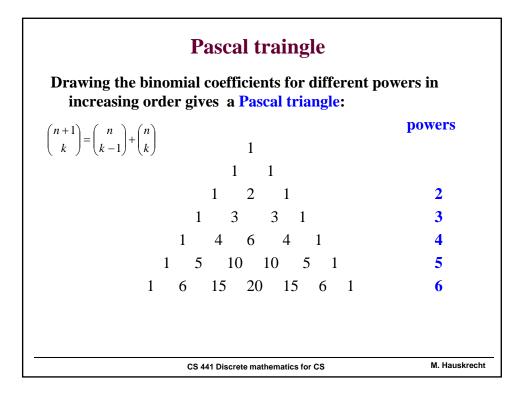


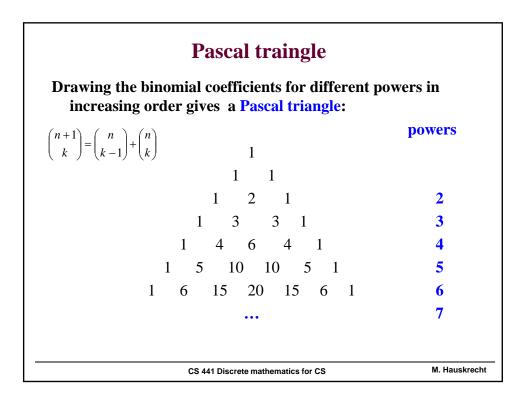


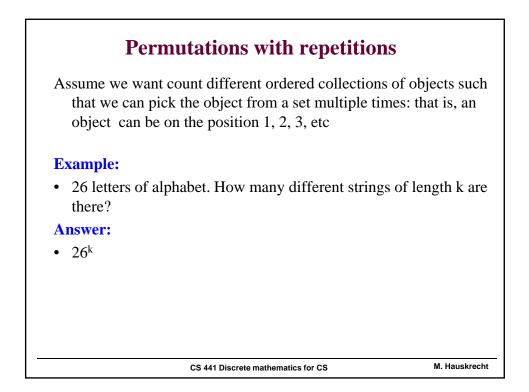


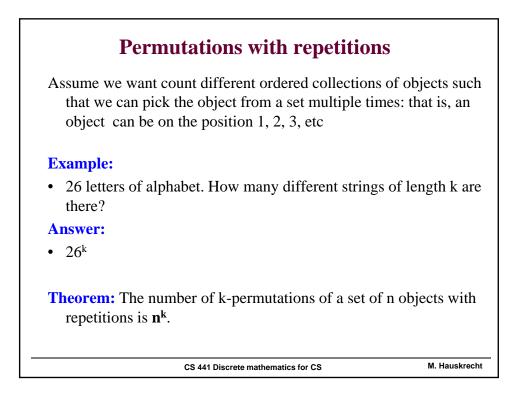


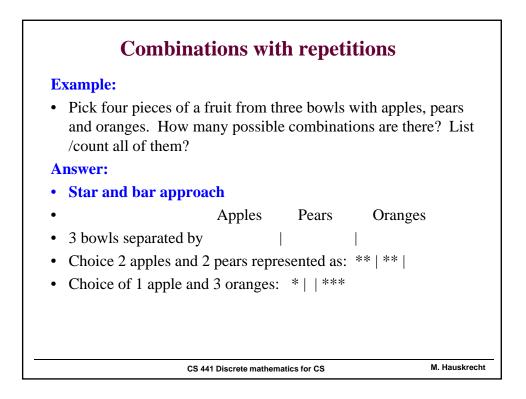




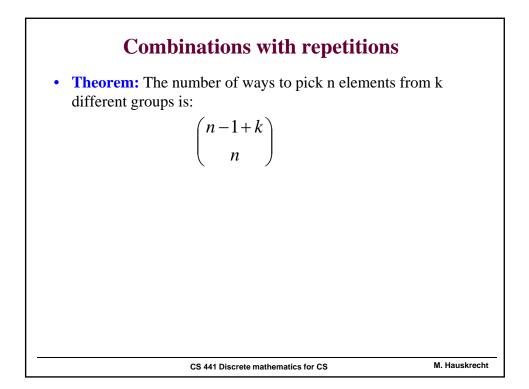


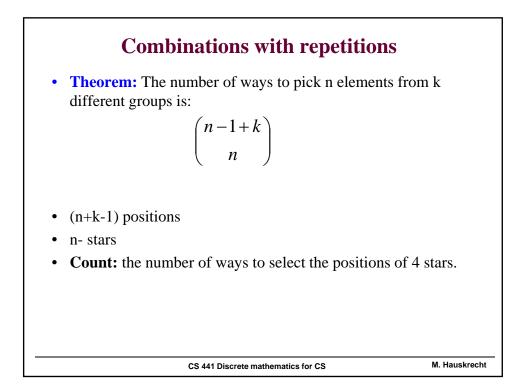


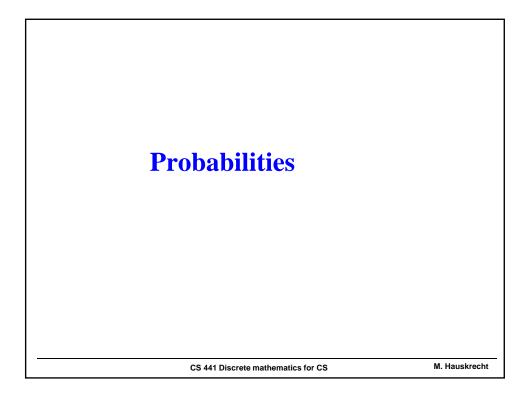




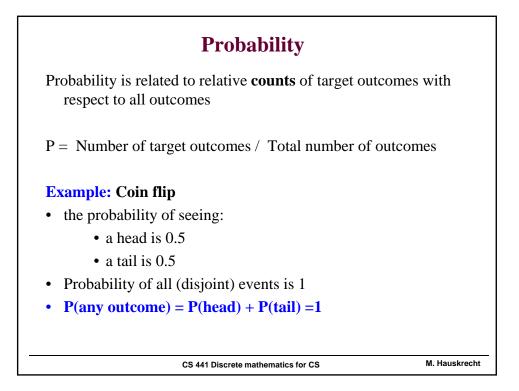
Combinations with repetitions		
Example:		
• Pick four pieces of a fruit from three bowls with apples, pears and oranges. How many possible combinations are there? List /count all of them?		
Answer:		
Star and bar approach		
Apples Pears Oranges		
• 3 bowls separated by		
• Choice 2 apples and 2 pears represented as: ** **		
• Choice of 1 apple and 3 oranges: * ***		
• Count: How many different ways of arranging (3-1)=2 bars and 4 stars are there?		
• Total number of positions: 2+4=6		
• Count: the number of ways to select the positions of 4 stars.		
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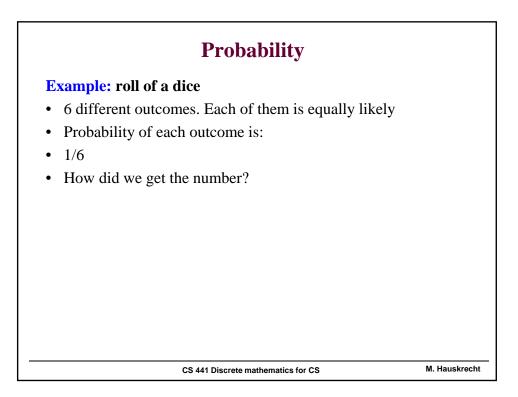


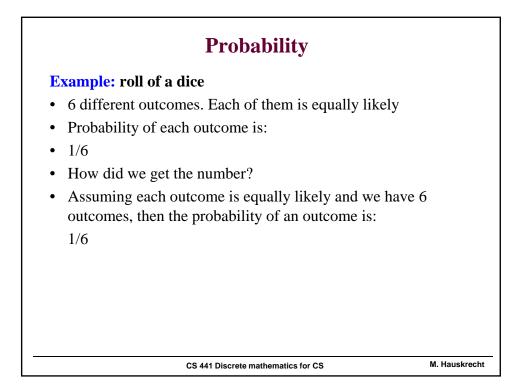




Probability		
Discrete probability theory		
• Dates back to 17th century.		
• It was used to compute the odds of seeing some outcomin games, races etc.	mes: e.g.	
• Odds are related to counting when the outcomes are equively.	qually	
Example: Coin flip		
• Assume 2 outcomes (head and tail) and each of them i likely	s equally	
• Odds: 50%, 50%		
• the probability of seeing:		
• a head is 0.5		
• a tail is 0.5		
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Probability of aggregate outcomes Example: roll of a dice Roll of the dice is odd or even. All outcomes are equally likely. Probability = number of outcomes when odd/ total number of outcomes. Solution 1: all outcomes are equally likely and = 1/6 Odd numbers: 1,3,5 Even numbers: 2,4,6 P(odd) = 1/6+1/6+1/6 = 3/6 = 1/2 P(even) = 3/6 = 1/2 Odd numbers are equally likely as even numbers (2 outcomes) P(odd) = 1/2 and P(even) = 1/2

