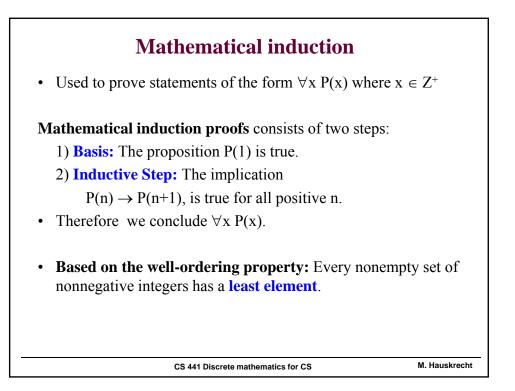
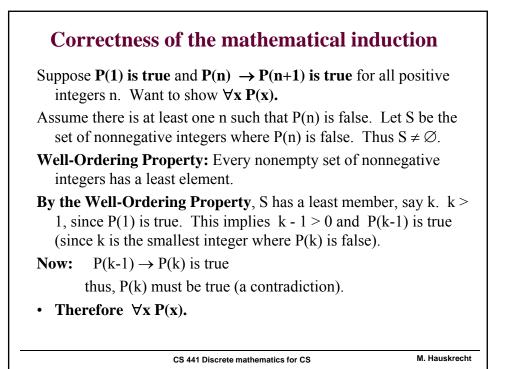


Proofs		
Basic proof methods:		
• Direct, Indirect, Contradiction, By Cases, Equivalences		
Proof of quantified statements:		
• There exists x with some property P(x).		
<ul> <li>It is sufficient to find one element for which the propert holds.</li> </ul>	ty	
• For all x some property P(x) holds.		
<ul> <li>Proofs of 'For all x some property P(x) holds' must cov x and can be harder.</li> </ul>	er all	
• Mathematical induction is a technique that can be applied prove the universal statements for sets of positive integers their associated sequences.		
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Mathematical induction		
<b>Example:</b> Prove the sum of first n odd integers is $n^2$ .		
i.e. $1 + 3 + 5 + 7 + + (2n - 1) = n^2$ for all positive in	tegers.	
Proof:		
• What is $P(n)$ ? $P(n)$ : $1 + 3 + 5 + 7 + + (2n - 1) =$	n <sup>2</sup>	
<b>Basis Step</b> Show P(1) is true		
• Trivial: $1 = 1^2$		
<b>Inductive Step</b> Show if $P(n)$ is true then $P(n+1)$ is true for	r all n.	
• Suppose P(n) is true, that is $1 + 3 + 5 + 7 + + (2n - 3)$	$1) = n^2$	
• Show P(n+1): 1 + 3 + 5 + 7 + + (2n - 1) + (2n + 1) = follows:	$=(n+1)^2$	
• $1+3+5+7++(2n-1)+(2n+1) =$		
$n^2$ + (2n+1) = (n+1)^2		
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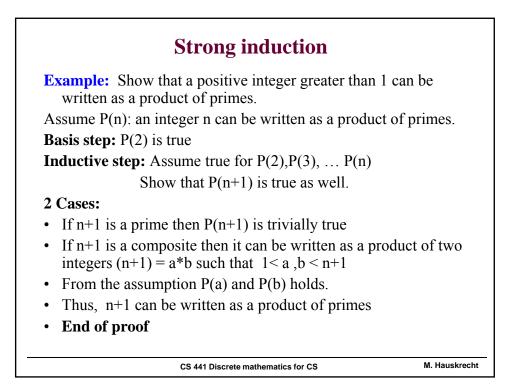


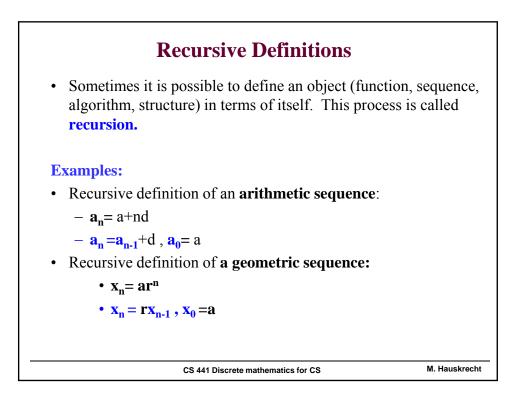
Mathematical induction		
<b>Example:</b> Prove $n < 2^n$ for all positive integers n. • P(n): $n < 2^n$		
Basis Step: $1 < 2^1$ (obvious) Inductive Step: If P(n) is true then P(n+1) is true for each • Suppose P(n): $n < 2^n$ is true • Show P(n+1): $n+1 < 2^{n+1}$ is true. $n + 1 < 2^n + 1$ $< 2^n + 2^n$ $= 2^n (1 + 1)$ $= 2^n (2)$ $= 2^{n+1}$	ch n.	
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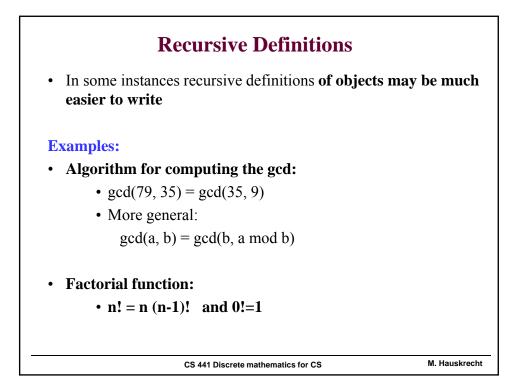
## Mathematical induction

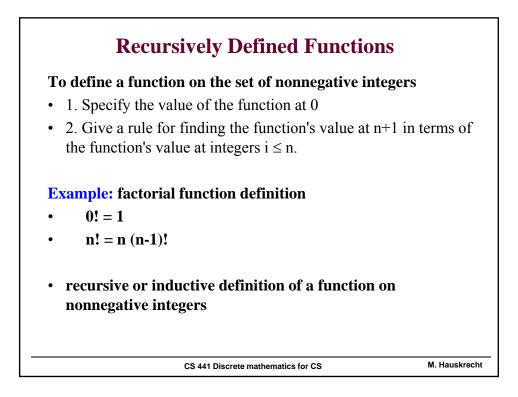
**Example:** Prove  $n^3 - n$  is divisible by 3 for all positive integers. • P(n):  $n^3 - n$  is divisible by 3 **Basis Step:** P(1):  $1^3 - 1 = 0$  is divisible by 3 (obvious) **Inductive Step:** If P(n) is true then P(n+1) is true for each positive integer. • Suppose P(n):  $n^3 - n$  is divisible by 3 is true. • Show P(n+1):  $(n+1)^3 - (n+1)$  is divisible by 3.  $(n+1)^3 - (n+1) = n^3 + 3n^2 + 3n + 1 - n - 1$ = (n<sup>3</sup> - n) + 3n<sup>2</sup> + 3n  $= (n^3 - n) + 3(n^2 + n)$ 尽 ス divisible by 3 divisible by 3 M. Hauskrecht CS 441 Discrete mathematics for CS

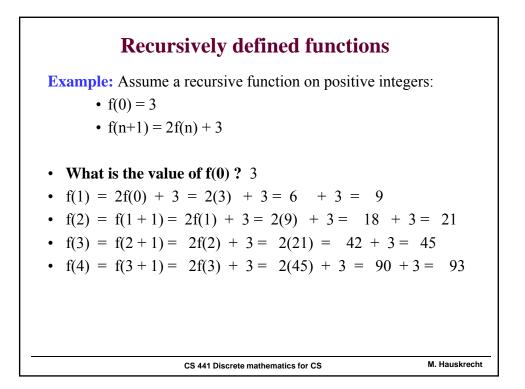
<section-header>Strong induction• The regular induction• uses the basic step P(1) and• inductive step P(n-1) → P(n)• Strong induction uses:• Uses the basis step P(1) and• inductive step P(1) and P(2) ... P(n-1) → P(n)• Example: Show that a positive integer greater than 1 can be written as a product of primes.











<b>Recursive definitions</b>		
• Example: Define the function: f(n) = 2n + 1 $n = 0, 1, 2,recursively.$		
• $f(0) = 1$ • $f(n+1) = f(n) + 2$		
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