

Problem assignment 3

Due: Wednesday, January 31, 2007

Problem 1. Poisson distribution

The Poisson distribution is used to model the number of random arrivals to a system over a fixed period of time. Examples of systems in which events are determined by random arrivals are: arrivals of customers requesting the service, occurrence of natural disasters, such as floods, etc. The Poisson distribution is defined as:

$$p(x|\lambda) = \frac{e^{-\lambda} \lambda^x}{x!} \text{ for } x = 0, 1, 2, \dots$$

Answer the following questions:

- (a) Using the definition of the Poisson distribution show that the sum of probabilities of all events is 1. (Hint: use the definition of e^λ in terms of a sum).
- (b) Derive the mean of the Poisson distribution.
- (c) Assume we have n independent samples of x . What is the ML estimate of the parameter λ .
- (d) The conjugate prior for the Poisson distribution is Gamma distribution. It is defined as:

$$p(\lambda|a, b) = \frac{1}{b^a \Gamma(a)} \lambda^{(a-1)} e^{-\frac{\lambda}{b}}.$$

Show that the posterior density of the parameter λ is again a Gamma distribution.

- (e) Show that the Poisson distribution is a member of the exponential family of distributions. Give η , $T(x)$, $Z(\eta)$ and $h(x)$ components.

Now we are ready to do some Matlab experiments:

- (f) plot the probability function for Poisson distributions with parameters $\lambda = 2$ and $\lambda = 6$. Note that the Poisson model is defined over nonnegative integers only.

- (g) Assume the data in 'poisson.txt' that represent the number of incoming phone calls received over a fixed period of time. Compute and report the ML estimate of the parameter λ .
- (h) Assume the prior on λ is given by $\lambda \sim \text{Gamma}(a, b)$. Plot the Gamma distribution for the following set of parameters ($a = 1, b = 2$) and ($a = 3, b = 5$).
- (g) Plot the posterior density for λ after observing samples in 'poission.txt' and using priors in part (h). What changes in the distribution do you observe?

Note: it is not necessary to submit any Matlab code for this assignment, just include the plots in your report.