

Problem assignment 6

Due: Wednesday, November 12, 2008

Probability theory

Problem 1

Let A,B,C be random variables. A and B have two possible values T,F and C has three values: high, medium, low. The full joint distribution is defined by the following contingency table.

| C / (A,B) | (T T) | (T F) | (F T) | (F F) |
|-----------|-------|-------|-------|-------|
| high | 0.02 | 0.08 | 0.03 | 0.12 |
| medium | 0.1 | 0.1 | 0.05 | 0.15 |
| low | 0.05 | 0.05 | 0.15 | 0.1 |

Calculate the following **probability distributions**:

- Part a. $\mathbf{P(A, B)}$.
- Part b. $\mathbf{P(A, C)}$.
- Part c. $\mathbf{P(B)}$.
- Part d. $\mathbf{P(A, B|C = \text{medium})}$.
- Part e. $\mathbf{P(A|B = T, C = \text{high})}$.

Problem 2

Random variables A,B are conditionally independent given C when:

$$P(A, B|C) = P(A|C)P(B|C).$$

Prove that this implies:

$$P(A|B, C) = P(A|C).$$

Problem 3

The probability it rains or snows in Boston is 0.1. The probability that the traffic on I-93 (an interstate passing through Boston) is slow during precipitation is 0.8. The probability the traffic is slow during normal weather (no rain or snow) is 0.3. Compute the probability of an actual weather pattern in Boston given the traffic on I-93 is slow.