University of Pittsburgh
CS 2740 Knowledge Representation — Handout 4
Professor Milos Hauskrecht — September 24, 2008

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## Problem assignment 3

*Due: Wednesday, October 1 2008*

### Problem 1. Inference with propositional rules.

Assume a simplified animal identification problem due to P. Winston. The knowledge needed for the problem consists of the following set of rules:

1. If the animal has hair then it is a mammal
2. If the animal gives milk then it is a mammal
3. If the animal has feathers then it is a bird
4. If the animal flies and it lays eggs then it is a bird
5. If the animal is a mammal and it eats meat then it is a carnivore
6. If the animal is a mammal and it has pointed teeth and it has claws and its eyes point forward then it is a carnivore
7. If the animal is a mammal it has hoofs then it is an ungulate
8. If the animal is a mammal and it chews cud then it is an ungulate
9. If the animal is a mammal and it chews cud then it is even-toed
10. If the animal is a carnivore and it has a tawny color and it has dark spots then it is a cheetah
11. If the animal is a carnivore and it has a tawny color and it has black strips then it is a tiger
12. If the animal is an ungulate and it has long legs and it has a long neck and it has a tawny color and it has dark spots then it is a giraffe
13. If the animal is an ungulate and it has a white color and it has black stripes then it is a zebra
14. If the animal is a bird and it does not fly and it has long legs and it has a long neck and it is black and white then it is an ostrich,
15. If the animal is a bird and it does not fly and it swims and it is black and white then it is a penguin
16. If the animal is a bird and it is a good flyer then it is an albatross.

The above set of rules can be represented in the propositional logic using implications of the form $A1 \wedge A2 \wedge \cdots \wedge Ak \rightarrow B$, that is, all the statements are in the Horn form. Recall that inferences with modus ponens for KB in the Horn normal form are both sound and complete.

### Part a. Inference with propositional rules

Assume a set of initial facts: the animal gives milk, it chews cud, it has long legs, long neck, tawny cloor and dark spots are all TRUE for the animal we want to identify. Assume the following set of theorems:

- Theorem1: the animal is a giraffe;
- Theorem2: the animal is a penguin;
- Theorem3: the animal is a mammal.

Decide using the repeated application of the modus ponens inference rule whether Theorems 1-3 hold. For every theorem proved give a sequence of rules (their numbers) used to derive the conclusion.

### Implementation of logical inference with propositional rules

Two procedures for making logical inferences with KBs in HNF are: forward and backward chaining. The procedures are sound and complete with respect to atomic propositions one can derive from the KB in the HNF.

### Part b. Forward chaining of propositional rules

Write a **forward-chaining** function that takes a knowledge base (with rules and initial facts) and returns a list of all atomic propositions it can derive from the knowledge base. The KB that is an argument to the function is a list of facts and rules. For example, a KB with facts $A, B$ and rules $A \wedge B \rightarrow C$ and $A \rightarrow D$ is represented as a list:

KB= '(A B ((A B) C) ((A) D)).

The output should be a list of all new facts the procedure will be able to derive. For the above example, the output of the procedure should be a list (C D).

The procedure can be implemented in a number of different ways and some solutions can be more efficient than others (see lecture for the example of an efficient implementation). One such implementation is described in Russell and Norvig textbook and was discussed during the lecture. For the purpose of this assignment, you do not have to implement the procedure that is the most efficient, it is more important to understand the main principle. Please include your forward chaining procedure in file $problem3 - 2b.lisp$.

### Part c. Backward chaining of propositional rules

The forward chaining procedure keep infering facts one can derive from the existing KB. The procedure is that are not helping in proving the target theorem. An alternative inference technique, called backward chaining, alleviates this difficulty.

The backward chaining proofs the theorem backwards: it starts from the theorem. It checks all rules with antecedents equal to the theorem to see whether their antecedents are satisfied. If at least one of the rules is satisfied the fact is proved and added to the fact base. If not, the facts that are not known in the rule premise become new theorems to be proved and the backward chaining procedure is called recursively on new theorems. Note that there may be many rules with the same antecedent so before saying that the fact cannot be proved make sure that all rules are exhausted and cannot be used to prove the theorem.

Write a **backward-chaining** procedure that takes a knowledge base as its first argument and the theorem to be proved as the second argument. It returns true if the theorem can be proved and false if it cannot. Please include the code of your forward chaining procedure in the file $problem3-2c.lisp$.

## Problem 2. First-order logic

Express the following sentences in the first-order logic. Assume that the universe of discourse for people are people.

a. Some students took both History and Biology class in Spring 2002.
b. At least one student failed History.
c. At least one student failed both History and Biology.
d. All students who took History also took Biology.
e. Every person who buys an insurance policy is smart.
f. No person buys an expensive insurance policy.
g. There is a woman who likes all men who are not vegetarians.
h. There is a barber who shaves all men in town who do not shave themselves.
i. No person likes a professor unless the professor is smart.
j. Politicians can fool all of the people some of the time, but they can't fool all of the people all of the time.