

Problem assignment 2

Due: Wednesday, September 24, 2000

Propositional logic

Problem 1

Determine whether the two expressions are logically equivalent, that is, for each interpretation they lead to the same value. Hint: use truth table to show the equivalence.

- $\neg(A \wedge B)$ and $(\neg A \vee \neg B)$
- $(P \rightarrow R)$ and $(\neg R \rightarrow \neg P)$
- $(P \rightarrow R)$ and $(\neg P \rightarrow \neg R)$
- $(P \rightarrow R) \wedge (Q \rightarrow R)$ and $(P \vee Q) \rightarrow R$
- $(P \rightarrow R) \wedge (P \rightarrow Q)$ and $P \rightarrow (Q \vee R)$

Problem 2.

Let KB consists of the following sentences:

$$\begin{aligned} &\neg(R \wedge \neg P) \vee \neg(\neg S \wedge \neg T), \\ &\qquad\qquad\qquad \neg(T \vee P), \\ &Q \rightarrow (\neg T \rightarrow (\neg S \wedge R)). \end{aligned}$$

Prove that $\neg Q$ holds using:

- **Part a.** Truth-table approach.
- **Part b.** Inference rule approach.
- **Part c.** Resolution with refutation.

Problem 3. Implementation of the resolution-refutation solver.

Resolution refutation offers one method for solving the logical inference problem. The method works with the Knowledge base (KB) that has been transformed into the conjunctive normal form (CNF). Briefly, the CNF permits the conjunction of clauses. Each clause consists of a disjunction of literals, where literals are either negated or non-negated propositional symbols. The theorem is checked for entailment by first converting the negated theorem to the CNF and then checking if it contradicts with the KB. Essentially, the contradiction-checking part works with a logical expression in the CNF that is obtained by merging together CNFs for the KB and for the negated theorem.

Write **resolution-refutation** function that takes as an argument a list that represents a logical expression written in the CNF, and decides using the resolution rule whether a contradiction can be inferred or not. The function should return NIL if the contradiction was not found, otherwise a propositional symbol that led to the contradiction should be returned.

Assume the CNF expressions are represented by a structured list, where the top-level list represents conjunction of clauses and the second level lists clauses (disjunctions of literals). The negated propositional symbols are represented as a list: (not A). For example, the following CNF expression should be translated as follows:

$$(A \vee B \vee \neg C) \wedge (\neg A \vee D) \wedge \neg B$$

translates to:

(and (or A B (not C)) (or (not A) D) (or (not B)))

After thoroughly testing the function, please include it in problem2-3.lisp file and submit.