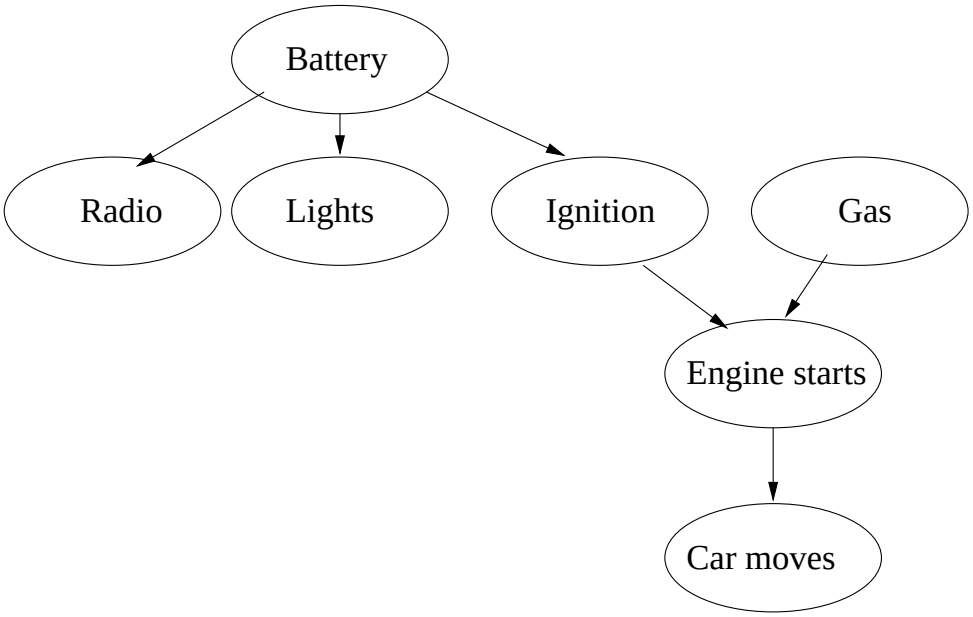
Problem set 8 solutions

Uncertainty.

Problem 1. Monte Carlo sampling

Assume the Bayesian belief network for the diagnosis of car's electrical system.



Part a. Briefly explain how would you calculate the probability $P(Ignition|Carmoves = F, Radio = T)$ using rejection sampling. What is the deficiency of the method.

The rejection sampling generates samples randomly from the full joint distribution. In BBN this is achieved by sampling the nodes (variables) according to the node order defined by links, starting from nodes with no parents. The samples that do not satisfy the condition ($Carmoves = F$ and $Radio = T$) are then excluded. The limitation of the methods is that samples not consistent with the condition are wasted.

Part b. Briefly explain how would you calculate the probability $P(Ignition|Carmoves = F, Radio = T)$ using the likelihood weighting approach.

Answer: The samples in the likelihood weighting approach are generated such that all nodes (variables) defining the condition are fixed, the remaining nodes are sampled according to the conditional distribution associated with these nodes. Similarly to forward sampling, the sampling process respects the direction of links, such that all parent nodes get the value before their descendant nodes.

Part c. Assume that the likelihood weighting sampling process for computing $P(Ignition|Carmoves = F, Radio = T)$ generated the following example: $Battery = T, Radio = T, Ignition = T, Light = T, EngineStatus = Fail, Gas = T, Carmoves = F$. Show how to calculate the weight associated with this example. Your formula should use the conditional probabilities that are defined by the BBN.

Answer: The above example receives the following weight:

$$P(Radio = T|Battery = T)P(Carmoves = F|EngineStatus = Fail).$$

Problem 2. Investments.

Assume you have to invest $10K$ for 2 investment periods. Your options are the stock market and the bank. The probability of a stock going up in the first period is: 0.4. The probability of a stock going up in the second period depends on the first period stock outcomes and equal: $P(2nd = up|1st = up) = 0.3$ and $P(2nd = up|1st = down) = 0.5$. The monetary returns for different scenarios are defined as follows:
(A1: stock, S1: up, A2: stock, S2: up): 19K
(A1: stock, S1: up, A2: stock, S2: down): 12.5K
(A1: stock, S1: up, A2: bank, S2: any): 14.5K
(A1: stock, S1: down, A2: stock, S2: up): 10K
(A1: stock, S1: down, A2: stock, S2: down): 6K
(A1: stock, S1: down, A2: bank, S2: any): 8K
(A1: bank, S1: any, A2: stock, S2: up): 13.5K
(A1: bank, S1: any, A2: stock, S2: down): 8K
(A1: bank, S1: any, A2: bank, S2: any): 10.5K
where A1 denotes action 1, S1 the movement of the stock in period 1, A2 action 2 and S2 the movement of the stock in period 2.

Please draw a decision tree corresponding to the above investment problem. Use the tree to calculate the optimal investment plan for the two investment periods.

Answer: This is the decision tree. The rational course of action is to invest in stock in the first period. If the stock market goes up, go with the bank in the second period. If it goes down, both decisions are equally good. This behavior maximizes the expected monetary value of the

outcome and is risk-neutral.