University of Pittsburgh
CS 2740 Knowledge Representation
Professor Milos Hauskrecht

Handout 9
October 15, 2008

Solutions to problem set 4

Problem 1. Unification

See example solution on the course web site.

Problem 2. Inference in FOL

Let L be a first-order language that contains the predicates, A(x), C(x), D(x), to say x is an animal, cat, and dog, respectively, and L(x; y) and F(x; y), to say x loves y and y is a friend of x, respectively.

Part a. Translate the following knowledge base into the FOL.

1. Cats and dogs are animals.
   Translation: $\forall x(C(x) \rightarrow A(x)) \wedge (D(x) \rightarrow A(x))$
2. Everyone loves either a cat or a dog.
   Translation: $\forall y \exists z(C(z) \vee D(z)) \wedge L(y, z)$
3. Anyone who loves an animal has a friend.
   Translation: $\forall u(\exists v L(u, v) \wedge A(v)) \rightarrow \exists t F(u, t)$
4. Everyone has a friend.
   Translation: $\forall x \exists y F(x, y)$

Part b. Convert these formulas into normal form, negating the last beforehand.

- $\forall x(C(x) \rightarrow A(x)) \wedge \forall x(D(x) \rightarrow A(x))$
  $(\neg C(x) \vee A(x)) \wedge (\neg D(q) \vee A(q))$
- $\forall y \exists z(C(z) \vee D(z)) \wedge L(y, z)$
  $[C(f_1(y)) \vee D(f_1(y))] \wedge L(y, f_1(y))$, where $f_1(y)$ is a Skolem function
- $\forall u(\exists v L(u, v) \wedge A(v)) \rightarrow \exists t F(u, t)$
  $(\neg L(u, v) \vee \neg A(v) \vee F(u, f_2(u))$, where $f_2(u)$ is a Skolem function

- $\neg \forall x \exists y F(x, y)$
  $\neg F(K, w)$, where K is a Skolem constant

Part c. Prove that everyone has a friend (sentence 4) follows from sentences 1,2 and 3, using generalized resolution and proof-by-refutation.

- 1. $\neg C(x) \vee A(x)$
- 2. $\neg D(q) \vee A(q)$
- 3. $C(f_1(y)) \vee D(f_1(y))$
- 4. $L(y, f_1(y))$
- 5. $\neg L(u, v) \vee \neg A(v) \vee F(u, f_2(u))$
- 6. $\neg F(K, w)$
- 7. $\neg L(K, v) \vee \neg A(v)$ from 5 & 6 $(u/K, w/f_2(K))$
- 8. $\neg A(f_1(K))$ from 7 & 8 $(y/K, v/f_1(K))$
- 9. $\neg C(f_1(K))$ from 8 & 1 $(x/f_1(K))$
- 10. $\neg D(f_1(K))$ from 8 & 2 $(q/f_1(K))$
- 11. $D(f_1(K))$ from 9 & 3 (y/K)
- 12. contradiction, from 10 & 11