Solutions to problem set 3

Problem 1. Inference with propositional rules.

Assume a simplified animal identification problem due to P. Winston. The knowledge needed for the problem consists of the following set of rules:

1. If the animal has hair then it is a mammal
2. If the animal gives milk then it is a mammal
3. If the animal has feathers then it is a bird
4. If the animal flies and it lays eggs then it is a bird
5. If the animal is a mammal and it eats meat then it is a carnivore
6. If the animal is a mammal and it has pointed teeth and it has claws and its eyes point forward then it is a carnivore
7. If the animal is a mammal it has hoofs then it is an ungulate
8. If the animal is a mammal and it chews cud then it is an ungulate
9. If the animal is a mammal and it chews cud then it is even-toed
10. If the animal is a carnivore and it has a tawny color and it has dark spots then it is a cheetah
11. If the animal is a carnivore and it has a tawny color and it has black strips then it is a tiger
12. If the animal is an ungulate and it has long legs and it has a long neck and it has a tawny color and it has dark spots then it is a giraffe
13. If the animal is an ungulate and it has a white color and it has black stripes then it is a zebra
14. If the animal is a bird and it does not fly and it has long legs and it has a long neck and it is black and white then it is an ostrich,
15. If the animal is a bird and it does not fly and it swims and it is black and white then it is a penguin
16. If the animal is a bird and it is a good flyer then it is an albatross.

The above set of rules can be represented in the propositional logic using implications of the form $A1 \wedge A2 \wedge \cdots \wedge Ak \rightarrow B$, that is, all the statements are in the Horn form. Recall that inferences with modus ponens for KB in the Horn normal form are both sound and complete.

Part a. Assume a set of initial facts: the animal gives milk, it chews cud, it has long legs, long neck, tawny cloor and dark spots are all TRUE for the animal we want to identify. Assume the following set of theorems:

- Theorem1: the animal is a giraffe;
- Theorem2: the animal is a penguin;
- Theorem3: the animal is a mammal.

Decide using the repeated application of the modus ponens inference rule whether Theorems 1-3 hold. For every theorem proved give a sequence of rules (their numbers) used to derive the conclusion.

Answer.

- Theorem1: the animal is a giraffe is TRUE. The sequence of rules to prove this: R2, R8, R12.
- Theorem2: the animal is a penguin cannot be proved. None of the conditions of rule R15 can be proved.
- Theorem3: the animal is a mammal is TRUE. The sequence of rules to prove this is: R2.

  Parts b and c. See the programming solution on the course web page.

Parts b and c

See the course web page for the solutions to these parts.

Problem 2. FOL: Translation

Express the following sentences in the first-order logic.

a. Some students took both History and Biology.
$\exists x Student(x) \wedge (Takes(x, H) \wedge Takes(x, B))$.
b. At least one student failed History.
$\exists x Student(x) \wedge Fails(x, H)$
c. At least one student failed both History and Biology.
$\exists x Student(x) \wedge Fails(x, H) \wedge Fails(x, B)$
d. All student who took History also took Biology.
$\forall x (Student(x) \wedge Takes(x, H)) \rightarrow Takes(x, B)$.
e. Every person who buys an insurance policy is smart.

$\forall x Person(x) \wedge (\exists y Policy(y) \wedge Buys(x, y)) \implies Smart(x)$.

f. No person buys an expensive policy.

$\forall x, y Person(x) \wedge Policy(y) \wedge Expensive(y) \implies \neg Buys(x, y)$

g. There is a woman who likes all men who are not vegetarians.

$\exists x Woman(x) \wedge \forall y Man(y) \wedge \neg Vegetarian(y) \implies Likes(x, y)$.

h. There is a barber who shaves all men in town who do not shave themselves.

$\exists x Barber(x) \wedge \forall y Man(y) \wedge \neg Shaves(y, y) \implies Shaves(x, y)$.

i. No person likes a professor unless the professor is smart.

$\forall x, y Person(x) \wedge Professor(y) \wedge \neg Smart(y) \implies \neg Likes(x, y)$.

j. Politicians can fool can fool all of the people some of the time, but they can't fool all of the people all of the time

$\forall x Politician(x) \implies (\forall y \exists t Person(y) \implies Fools(x, y, t)) \wedge \neg (\forall y \forall t Person(y) \implies Fool(x, y, t))$.

Other possibility (also correct) is to consider a constant Politicians denoting a group of people:

$(\forall y \exists t Person(y) \implies Fools(Politicians, y, t)) \wedge \neg (\forall y \forall t Person(y) \implies Fools(Politicians, y, t))$.

There is an ambiguity in the meaning of 'they can fool all of the people some of the time' and both of the following translations are correct:

$(\forall y \exists t Person(y) \implies Fools(x, y, t))$

$(\exists t \forall y Person(y) \implies Fools(x, y, t))$.