University of Pittsburgh
CS 2740 Knowledge Representation — Handout 7
Professor Milos Hauskrecht — October 8, 2008

Solutions to problem set 2

Problem 1. Equivalences

Determine whether the two expressions are logically equivalent, that is, for each interpretation they lead to the same value. Hint: use truth table to show the equivalence.

- $\neg(A \wedge B)$ and $(\neg A \vee \neg B)$

A	B	$\neg(A \wedge B)$	$(\neg A \vee \neg B)$
1	1	0	0
1	0	1	1
0	1	1	1
0	0	1	1

- $(P \rightarrow R)$ and $(\neg R \rightarrow \neg P)$

P	R	$(P \rightarrow R)$	$(\neg R \rightarrow \neg P)$
1	1	1	1
1	0	0	0
0	1	1	1
0	0	1	1

- $(P \rightarrow R)$ and $(\neg P \rightarrow \neg R)$

P	R	$(P \rightarrow R)$	$(\neg P \rightarrow \neg R)$
1	1	1	1
1	0	0	1
0	1	1	0
0	0	1	1

  not equivalent

- $(P \rightarrow R) \wedge (Q \rightarrow R)$ and $(P \vee Q) \rightarrow R$

P	R	Q	$(P \rightarrow R)$	$Q \rightarrow R$	$(P \rightarrow R) \wedge Q \rightarrow R$	$(P \vee Q) \rightarrow R)$
1	1	1	1	1	1	1
1	1	0	1	1	1	1
1	0	1	0	0	0	0
1	0	0	0	1	0	0
0	1	1	1	1	1	1
0	1	0	1	1	1	1
0	0	1	1	0	0	0
0	0	0	1	1	1	1

- $(P \rightarrow R) \wedge (P \rightarrow Q)$ and $P \rightarrow (Q \vee R)$

P	R	Q	$(P \rightarrow R)$	$P \rightarrow Q$	$(P \rightarrow R) \wedge (P \rightarrow Q)$	$P \rightarrow (Q \vee R)$
1	1	1	1	1	1	1
1	1	0	1	0	0	0
1	0	1	0	1	0	0
1	0	0	0	0	0	0
0	1	1	1	1	1	1
0	1	0	1	1	1	1
0	0	1	1	1	1	1
0	0	0	1	1	1	1

Problem 2. Inference in propositional logic

Let KB consists of the following sentences:

$$\begin{aligned} \neg(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T), \\ \neg(T \vee Q), \\ U \rightarrow (\neg T \rightarrow (\neg S \wedge P)). \end{aligned}$$

Prove that $\neg U$ holds using resolution with refutation.

Part a. Truth-table approach.

Answer. The complete truth table for the problem is:

U	P	Q	S	T	$(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T)$	$\neg(T \vee Q)$	$U- > (\neg T- > (\neg S \wedge P))$	KB	KB true mark
1	1	1	1	1	1	0	0	0	
1	1	1	1	0	1	0	0	0	
1	1	1	0	1	1	0	1	0	
1	1	1	0	0	0	0	1	0	
1	1	0	1	1	1	0	0	0	
1	1	0	1	0	1	1	0	0	
1	1	0	0	1	1	0	1	0	
1	1	0	0	0	1	1	1	0	
1	1	1	1	1	1	0	0	0	
1	0	1	1	0	1	0	0	0	
1	0	1	0	1	1	0	0	0	
1	0	1	0	0	0	0	0	0	
1	0	0	1	1	1	0	0	0	
1	0	0	1	0	1	1	0	0	
1	0	0	0	1	1	0	0	0	
1	0	0	0	0	0	1	0	0	
0	1	1	1	1	1	0	1	0	
0	1	1	1	0	1	0	1	0	
0	1	1	0	1	1	0	1	0	
0	1	1	0	0	0	0	1	0	
0	1	0	1	1	1	0	1	0	
0	1	0	1	0	1	1	1	1	*
0	1	0	0	1	1	0	1	0	
0	1	0	0	0	1	1	1	1	*
0	1	1	1	1	1	0	1	0	
0	0	1	1	0	1	0	1	0	
0	0	1	0	1	1	0	1	0	
0	0	1	0	0	0	0	1	0	
0	0	0	1	1	1	0	1	0	
0	0	0	1	0	1	1	1	1	*
0	0	0	0	1	1	0	1	0	
0	0	0	0	0	0	1	1	0	

The table may be interpreted in two interesting ways.

a) Assignment of True to U makes KB False - proof by refutation.

b) Everywhere KB is true, $\neg U$ is also true - $KB \models \neg U$

Part b. Inference rule approach.

Answer. These are the axioms:

1. $\neg(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T)$
2. $\neg(T \vee Q)$
3. $U- > (\neg T- > (\neg S \wedge P))$

This is a possible inference sequence:

4. $\neg P \vee Q \vee S \vee T$, DeMorgan on 1
5. $\neg U \vee T \vee (\neg S \wedge P)$, from 3 and definition of $\rightarrow$
6. $(\neg U \vee T \vee \neg S) \wedge (\neg U \vee T \vee P)$, from 5 and distribution over $\wedge$

7. $\neg U \vee T \vee \neg S$, and-elimination on 6
8. $\neg U \vee T \vee P$, and-elimination on 6
9. $\neg T \wedge \neg Q$, DeMorgan on 2
10. $\neg T$, and-elimination on 9
11. $\neg Q$, and-elimination on 9
12. $\neg P \vee S \vee T$, unit resolution 4 and 11
13. $\neg P \vee S$, unit resolution 12 and 10
14. $\neg U \vee P$, unit resolution 8 and 10
15. $\neg U \vee \neg S$, unit resolution 7 and 10
16. $\neg U \vee S$, resolution 13, 14
17. $\neg U \vee \neg U$, resolution 15,16
18. $\neg U$, tautology 17 (**the theorem is proved !!**)

Part c. Resolution refutation

Answer. In the refutation proof we add the negation of the statement we want to prove (the theorem) into the KB. Resolution rule works with statements in CNF. Thus, the first step is to convert all the sentences in the KB and negated theorem to CNF. Conversions: Sentence 1: $\neg(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T) \equiv \neg P \vee Q \vee S \vee T$.

Sentence 2: $\neg(T \vee Q) \equiv \neg T \wedge \neg Q$.

Sentence 3:

$$\begin{aligned}
U \rightarrow (\neg T \rightarrow (\neg S \wedge P)) &\equiv U \rightarrow (\neg(\neg T) \vee (\neg S \wedge P)) \\
&\equiv U \rightarrow (T \vee (\neg S \wedge P)) \\
&\equiv \neg U \vee T \vee (\neg S \wedge P) \\
&\equiv (\neg U \vee T \vee \neg S) \wedge (\neg U \vee T \vee P).
\end{aligned}$$

Negated theorem: U

Combining the above results we have the following set of clauses:
(1)$\neg P \vee Q \vee S \vee T$.
(2)$\neg T$.
(3)$\neg Q$.
(4)$\neg U \vee T \vee \neg S$

(5)$\neg U \vee T \vee P$
(6) U

We can prove the theorem by finding the sequence of resolution rules that lead to a contradiction. Such a proof is shown below.

clause #	new clause	resolution applied to
(7)	$\neg U \vee T \vee \neg P \vee Q$	(4) & (1)
(8)	$\neg U \vee T \vee Q$	(7) & (5)
(9)	$\neg U \vee T$	(8) & (3)
(10)	$\neg U$	(9) & (2)
(11)	$\{\}$	(6) & (10)

This proves the contradiction and thus $\neg U$ is entailed by the KB.

Problem 3

See the solutions on the course web page.