CS 1511/2110 Midterm 1
Spring 2018

Directions

1. The test is closed book and closed notes.
2. There are 8 part B questions. Answer at most 6 part B questions. Please try to limit your answers to one sentence when possible. One sentence should be quite sufficient for most, but maybe not all, part B questions. Part B questions are worth 10 points per question.
3. There are 4 part A questions. Answer at most 2 part A questions. Part A questions are worth 20 points per question.
4. Time will likely be an issue for most students. So use time wisely.
5. In particular, for the part A questions, usually it is then a good idea for the start of your answer to define relevant terms, give an overview of the proof strategy/technique that you will use, and to explain the key ideas are. If there is sufficient time, you may then start filling in the details.

PART B Questions

1. (a) Define what it means for an axiomatization to be sound.
   (b) Define what it means for an axiomatization to be complete.
   (c) Explain how, borrowing the proof technique used in the proof of Godel's first incompleteness theorem, you could plausibly show that there is no complete and sound axiomatization of Pruhsian geometry. Note that this type of geometry doesn't really exist, I'm just testing whether you understand the technique used in the proof of Godel's first incompleteness theorem.

2. (a) Define the Kolmogorov complexity $K(x)$ of a string x.
   (b) Prove that there is no Turing machine that can compute $K(x)$ given a string x.

3. (a) Is TIME(n^3) known to be a subset of SPACE(n^3)? Justify your answer.
   (b) Give a language that is in the complexity class TIME(n^3) but not in the complexity class TIME(n^2).

4. (a) Define the complexity class SPACE($\log n$).
   (b) Prove SPACE($\log n$) is a subset of the complexity class P.

5. (a) Define the complexity class Σ_3^p.
   (b) Explain why if PSPACE $\subseteq \Sigma_3^p$ then PSPACE $\subseteq \Pi_3^p$.

6. (a) Formally define P/poly.
   (b) Prove or disprove that P/poly is in PSPACE

7. (a) Define the complexity class ZPP.
   (b) Define the complexity class co-RP.
   (c) Show ZPP is a subset of co-RP.

8. (a) Draw a Venn diagram showing all known inclusion relationships for the following complexity classes: Σ_2^p, Π_2^p, BPP, Σ_1^p, Π_1^p, PSPACE, P.
   (b) State which of the above complexity classes are known to have complete problems under polynomial time reductions.

PART A Questions

1. Prove that if NP is a subset of P/poly then Π_2^p is a subset of Σ_2^p.

2. Prove that Circuit Satisfiability is complete for NP under polynomial time Karp reductions. The language Circuit Satisfiability consists of combinatorial circuits with one bit outputs such that there exists and input the causes the circuit to output a 1. Put yourself in the situation of Cook in 1972, and assume that it NP was not previously known to have complete problems.

3. Prove that there is a language L in Σ_5^p whose circuit complexity is $\Omega(n^5)$.

4. Prove that the following two definitions of $TIME(T(n))$ are equivalent in the sense that they contain exactly the same languages.

   Definition 1: $TIME(T(n))$ is the set of all languages L such that there exists a Turing machine M such that (1) M accepts x iff $x \in L$ and (2) for all but finitely many x, M on x halts in $T(|x|)$ steps.

   Definition 2: $TIME(T(n))$ is the set of all languages L such that there exists a Turing machine N and a number b such that (1) N accepts x iff $x \in L$ and (2) for all x, N on x halts within $b * T(|x|)$ steps.