

CS 1511/2110 Midterm 1
Spring 2016

Directions

1. The test is closed book and closed notes.
2. There are 12 part B questions. Answer at most 9 part B questions. Please try to limit your answers to one sentence. Part B questions are worth 10 points per question.
3. There are 5 part A questions. Answer at most 2 part A questions. Part A questions are worth 30 points per question.
4. Time will likely be an issue for most students. So use time wisely. Initially concentrate on the main ideas, and then fill in details with any remaining time.
5. In particular, for the part A questions, usually it is then a good idea for the start of your answer to define relevant terms, give an overview of the proof strategy/technique that you will use, and to explain the key ideas are. After this, you may launch into details.

PART B Questions

1. State the Church-Turing thesis, and explain why it is generally believed to be true.
2. Someone gives you a program $IAMFAMOUS(P, I)$ that they claim will determine whether the program P halts on the input I . Give an example of a program P and input I on which the program $IAMFAMOUS$ will not be correct.
3. State Gödel's (first) incompleteness theorem, and explain how its proof is related to the Halting Problem.
4. Give an example of a language that can be accepted using log space but that can not be accepted using constant space.
5. For each of the following two sentences, replace "?" with the smallest function that will make the sentence true:
 - (a) $TIME(f(n))$ is subset of $SPACE(?)$
 - (b) $SPACE(f(n))$ is a subset of $TIME(?)$.
6. Define the language TQBF (referred to in lecture as just QBF), and state the complexity class this language is complete for.
7. Explain how to prove that a language L is complete for a complexity class C under a type R of reduction.
8. Draw a Venn diagram for the complexity classes BPP , RP , $coRP$ and ZPP . Make sure that areas that are known to be empty have zero area in your drawing. For example, it is known that the intersection of classes X and Y is empty, then in your drawing X and Y should not intersect.
9. Give an example of a decidable language that is in $P/poly$, but not in P .
10. If you pick a function f uniformly at random from all functions from $\{0, 1\}^n$ to $\{0, 1\}$, the smallest circuit that computes f will likely contain how many gates (ignoring multiplicative constants)?
11. According to the Karp-Lipton theorem what is the consequence if the Boolean satisfiability problem (SAT) can be solved by Boolean circuits with a polynomial number of logic gates.
12. Define the complexity class Π_3^P .

PART A Questions

1. Prove $TIME(n^2)$ is a strict subset of $TIME(n^3)$. That is, show that there is a language in $TIME(n^3)$ that is not in $TIME(n^2)$.
2. Let P be some property of languages. Further assume there is a Turing machine M_1 that accepts a language L_1 that has property P , and a Turing machine M_2 that accepts a language L_2 that does not have property P . Show by reduction from the Halting Problem that there is no Turing machine that takes as input a Turing machine M , and determines whether the language $L(M)$ accepted by M satisfies property P .
3. Assume a log-space reduction from a language A to a language B .

So more precisely, there is a Turing machine T with three tapes, a read only input tape, a read/write work tape, and a write-only output tape. T only uses \log of the input size many cells on the read/write work tape. Further T never backs up the tape head on the write only output tape, so the tape head on the write only tape either stays in position or moves to the right. The machine T has the property that a string x is in A if and only if the contents of the write tape when T ends computation on input x is in B .

Now show that if there is a log space Turing machine S that accepts B , then there is a log space Turing Machine U that accepts A .

4. A nondeterministic circuit has two inputs x and y . We say that C accepts x if and only if there exists a y such that $C(x, y) = 1$. The size of the circuit is measured as a function of the size of x . Let $NP/poly$ be the languages that are decided by polynomial size nondeterministic circuits. Show $BP \cdot NP \subseteq NP/poly$. Recall that $BP \cdot NP$ is the set of languages L that such that there exists a randomized polynomial-time Turing machine M such that for all x , with probability at least $2/3$ it is the case that the Boolean formula output by $M(x)$ is satisfiable if and only if $x \in L$.
5. Show that BPP is a subset of Σ_2^p .